Efficient Transient Simulation of Lossy Interconnect

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Abstract

The problem of transient simulation of lossy transmission lines is investigated in this paper. Two refinements are made to the existing convolution approach for the case of a single lossy line: analytical formulae are derived for the line's impulse-responses, and an accurate numerical convolution technique that utilises these formulae are devised. It is shown that a special case of lossy multiconductor lines can be decomposed into uncoupled lossy lines and linear memoryless networks, leading to a simple simulation algorithm. Simulation results on industrial circuits with single and multiconductor lossy lines are presented and compared with results obtained using lumped and pseudo-lumped approximations of lossy lines. The comparison indicates that the convolution technique with the above enhancements can be an order-of-magnitude faster than lumped and pseudo-lumped segmenting techniques for equivalent or better accuracy.

1 Introduction

The need to consider interconnect in digital circuits as lossy transmission lines has arisen in recent years, particularly for multi-chip modules (MCMs) [1, 2, 3, 4]. Increasing lengths of interconnect combined with the faster switching speeds of modern logic [5] have created a situation where interconnect time-of-flight (delay) have become comparable to signal transition times, giving rise to transmission-line effects such as reflection and overshoot (see Fig. 1). Such phenomena are capable of causing undesired switching. In addition, the small cross-sectional areas of typical high-performance interconnect ($2\mu m \times 11\mu m$, [2]) give rise to series loss and the additional phenomena of attenuation and dispersion[1] (see Fig. 2). Dispersion can increase rise and fall times, and attenuation can drop signal levels into illegal regions. In situations where several interconnect run close together (as in busses), capacitive and inductive coupling give rise to coupling noise, a potential source of undesired switching. Multiconductor (or coupled) transmission-line models need to be used to estimate coupling noise. The need for accurate and efficient techniques to simulate lossy transmission lines in conjunction with digital logic is addressed in this paper.

The simplest lossy transmission-line model (the "simple lossy line", henceforth) has constant resistance, capacitance and inductance per unit length (R, L and C) uniformly distributed over the length of the line. Other more accurate and complex models have R, L and C as changing functions of frequency ("frequency-varying models", henceforth). (Skin-effect, an important phenomenon in which R increases with frequency, requires frequency-varying models [6, 7].) While the simulation of frequency-varying models is an important problem, Deutsch et. al. [6] have shown that the simple lossy line is an adequate model for most of today's applications. This paper is confined to the simple lossy line.

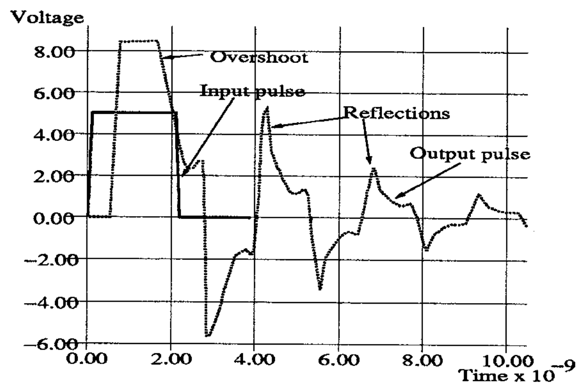

Figure 1: Reflection and Overshoot

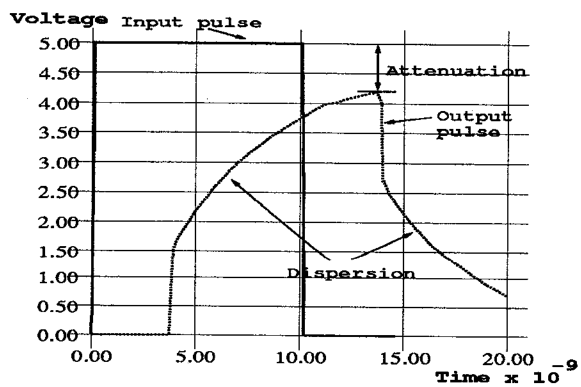

Figure 2: Dispersion and Attenuation

In the next section, previous work in lossy line simulation is briefly reviewed and the contributions of this paper stated.

[1]referring to the different propagation speeds of different frequency-components, a phenomenon that creates RC-type decays in waveforms

Sections 3, 4, and 5 deal with an improved convolution technique, analytical impulse responses, and multiconductor decomposition respectively. Simulation results on practical circuits are presented in Section 6, followed by concluding remarks and acknowledgments in Sections 7 and 8. Proofs and derivations are omitted for brevity and may be found in [8].

2 Previous Work, and Our Contributions

Transient simulation techniques for lossless transmission lines are well-known [9, 10, 11, 12, 13]. Several techniques exist for the transient simulation of lossy lines within circuits. The simplest and most prevalent is that of using segments to represent the line ("segmentation techniques"). In the lumped-RLC method [14], each segment is represented as a lumped RLC network, whereas in the pseudo-lumped method [15] a lossless line in series with a resistor is used. The pseudo-lumped method requires fewer segments than the lumped-RLC method and avoids the spurious oscillations that can appear in the latter; however, for both methods, a conservative number of segments is usually taken to ensure accuracy, and this can lead to large computation times. Fig. 3 shows the effects of varying the number of segments representing a line; computation time comparisons are presented in Section 6.

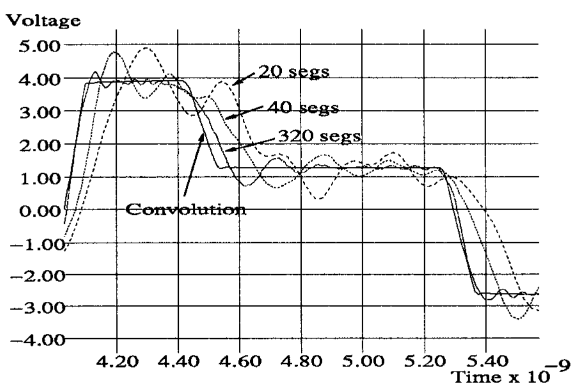

Figure 3: Lumped-RLC method, varying no. of segments

Completely linear circuits with lossy lines can be simulated entirely in the frequency domain [16, 7], using numerical Fourier transformation to transform inputs and outputs from and to the time domain. This technique is not applicable to circuits with nonlinear devices such as logic gates.

Waveform relaxation [17] has recently been applied to lossy-line simulation within nonlinear circuits. A mixed system of time and frequency domain circuit equations is solved by using each equation to update the waveform of one circuit variable while keeping all other variables fixed. Numerical Fourier transformation is used to switch between the time and frequency domain representations of each circuit variable. Speed and accuracy comparisons for this technique are not currently available.

In another approach (the "convolution approach"), the linearity of the lossy transmission line is exploited by expressing its outputs as a convolution of its inputs with impulse-responses specific to the line. Djordjević et. al. introduced this approach [18] and compared its performance with other methods [19], finding it slightly superior. Schutt-Aine and Mittra [20, 21] used a scattering-parameter approach to reformulate the transmission-line equations to obtain impulse-responses that died to zero rapidly.

Three components are crucial for the success of the convolution method: setting up the transmission-line equations properly, obtaining the line's impulse-responses accurately, and performing numerical convolution accurately within the circuit simulator. Previous work has concentrated mainly on the first component, using numerical FFTs[2] to obtain impulse responses and simple sample-and-sum approximations for convolution. In this work, three enhancements are made to the convolution approach:

1. An accurate technique for numerical convolution in the power series of the impulse-response is not truncated is described. The technique is a generalisation of the trapezoidal method for differential equations.

2. Exact analytical formulae are derived for the impulse-responses of the simple lossy line.

3. The special case of multiconductor simple lossy lines where all wires are electrically identical and each wire coupled only to its adjacent wires is shown to be decomposable into uncoupled simple lossy lines and linear memoryless networks. The decomposition leads directly to an efficient algorithm for the special case.

Enhancements 1 and 2 improve the accuracy of the technique and eliminate the need for numerical Fourier inversion. Enhancement 3 shows that solving a frequency-dependent eigenproblem [21] is not required, implying greater speed and accuracy of the proposed method for the special case.

In addition, a comparison of waveforms and simulation speeds of the lumped-RLC, pseudo-lumped and convolution methods is presented for typical industrial circuits and interconnect.

3 Numerical Convolution

In this section, an accurate method for performing convolution numerically is described. The convolution integral to be calculated is the following:

$$y(t) = \int_0^t x(\tau)\, h(t-\tau)\, d\tau \qquad (1)$$

In Equation 1, $x(\tau)$ is the input to the linear system described by the equation, $y(t)$ is the output and $h(\tau)$ is the impulse-response or the kernel of the system. $h(\tau)$ is assumed to be a constant well-known causal function of time and may be finite or infinite in duration. At any given time t, $x(\tau)$ and $y(\tau)$ are assumed to be known over the half-open interval $[0, t)$. It should be noted that in a circuit context, $x(\tau)$ and $y(\tau)$ are usually also related by a relation other than Equation 1; one may be a function or a causal functional of the other.

The problem of numerical convolution is the following:

> Given $x(t)$ and $y(t)$ related by Equation 1, and $x(t)$ *known only at a discrete number of time-points* $0,\ t_1\ ,\ t_2,\ \ldots,\ t_n$, *with* $t_i < t_{i+1}$, find $y(t_n)$. $h(t)$ is assumed to be well-known for all t.

[2]Fast Fourier Transform

Knowledge of the values of $x(t)$ at a discrete number of points is not sufficient to specify $y(t)$ uniquely by Equation 1. It is necessary to make assumptions about the nature of $x(t)$ such that its values at a discrete set of points suffice to specify $y(t)$ uniquely. In deriving linear multistep methods for differential equations [22, 23], the assumption that $x(t)$ is *piecewise-linear* results in the well-known *trapezoidal method* which is widely used in circuit simulators. The same assumption is made, and the numerical integration formula (Equation 2) is derived (see [8] for the derivation).

$$\begin{aligned}\int_0^{t_n} x(\tau)\, h(t_n-\tau)\, d\tau \;\approx\; & x_n \frac{F(t_n-t_{n-1})}{t_n-t_{n-1}} \\ & + \sum_{i=1}^{n-1} x_i \left[\frac{F(t_n-t_{i-1})-F(t_n-t_i)}{t_i-t_{i-1}}\right. \\ & \left. - \frac{F(t_n-t_i)-F(t_n-t_{i+1})}{t_{i+1}-t_i}\right] \quad (2)\end{aligned}$$

Here $x_i = x(t_i)$. $F(\cdot)$ is defined as follows:

$$F(t) = \int_0^t \int_0^\tau h(\tau')\, d\tau'\, d\tau \quad (3)$$

Strict equality holds in Equation 2 if the assumption that $x(t)$ is piecewise linear is valid; if not, the integration formula has an error term. This error is proportional to the second derivative of $x(t)$.

The stability of the numerical integration method depends on the nature of the kernel function $h(\tau)$ (more specifically, on $F(\tau)$). A theoretical proof of the stability of the numerical method for the impulse-responses of lossy transmission lines has not been found yet. The results of Section 6 indicate however that the numerical procedure is indeed stable.

4 Formulae for Impulse-Responses

In this section, the time-domain convolutive constitutive relationships for an uncoupled simple lossy line are derived. Analytical time-domain expressions for the voltage waveform in a lossy line with a step input have been presented in [1] when the far-end of the line is open, and in [24] for the terminated and infinite-line cases. Our contribution in this section is to present explicit expressions for the impulse-responses of a simple lossy line in a form suitable for direct application in the convolution algorithm described in Section 3.

The Telegrapher Equations for a lossy line are[21]:

$$\frac{\partial v}{\partial x} = -\left(L\frac{\partial i}{\partial t} + R\, i\right) \quad (4)$$

$$\frac{\partial i}{\partial x} = -\left(C\frac{\partial v}{\partial t}\, v\right) \quad (5)$$

In the above, the transmission line stretches from x coordinates 0 to l; $v(x,t)$ is the voltage at point x at time t; $i(x,t)$ is the current in the +ve x direction at x at time t. (The parallel conductance G is assumed to be zero for simplicity; all of the following continues to hold with minor modifications if it is nonzero.)

The boundary and initial conditions for Equations 4 and 5 are:

$$v(0,t) = v_1(t), \qquad v(l,t) = v_2(t) \quad (6)$$

$$i(0,t) = i_1(t), \qquad i(l,t) = -i_2(t) \quad (7)$$

$$v(x,0) = v_0(x), \qquad i(x,0) = i_0(x) \quad (8)$$

The object of the following is to establish time-domain relationships between v_1, v_2, i_1 and i_2 so that if any two are specified, the other two may be determined. Without loss of generality, it will be assumed that $v_1(t)$, $v_2(t)$, $i_1(t)$ and $i_2(t)$ are zero for all $t < 0$, and that $v_0(x)$ and $i_0(x)$ are identically zero. The assumption is made that all variables in Equations 4 and 5 are Laplace transformable in the time-domain and their Laplace transforms are taken to transform them into a pair of coupled ordinary differential equations in x and s (the Laplace variable). The coupling is removed by further differentiation and identical second-order ODEs obtained for the Laplace transforms of v and i. These equations are solved, and the boundary and initial conditions (Equations 6 - 8) applied to arrive at two equations relating the Laplace transforms of i_1, v_1, i_2 and v_2; these are in turn reformulated using a scattering-parameter approach to arrive at equations that are more suitable for numerical convolution. Finally, inverse Laplace transforms are taken to arrive at the time-domain equations. The details are presented in [8]. The time-domain convolution equations (constitutive relationships) finally obtained for the simple lossy line are the following:

$$v_1(t) * h_Y(t) - i_1(t) = v_2(t) * h_{\gamma Y}(t) + i_2(t) * h_\gamma(t) \quad (9)$$

$$v_2(t) * h_Y(t) - i_2(t) = v_1(t) * h_{\gamma Y}(t) + i_1(t) * h_\gamma(t) \quad (10)$$

where $*$ is the convolution operator. $h_Y(t)$, $h_\gamma(t)$ and $h_{\gamma Y}(t)$ are the three impulse-responses associated with the lossy transmission line. The expressions for these impulse-responses are:

$$h_Y(t) = Ye^{-\beta t}\left[\beta\left\{I_1(\beta t) - I_0(\beta t)\right\} + \delta(t)\right] \quad (11)$$

$$h_\gamma(t) = e^{-\beta t}\left[u_{t-T}\frac{\beta T}{\sqrt{t^2-T^2}} I_1(\beta\sqrt{t^2-T^2}) + \delta_{t-T}\right] \quad (12)$$

$$\begin{aligned}h_{\gamma Y}(t) = Ye^{-\beta t}\Big[u_{t-T}\,\beta\Big\{&\frac{t}{\sqrt{t^2-T^2}} I_1(\beta\sqrt{t^2-T^2}) \\ &- I_0(\beta\sqrt{t^2-T^2})\Big\} + \delta_{t-T}\Big] \quad (13)\end{aligned}$$

$$T = l\sqrt{LC}, \quad \beta = \frac{1}{2}\frac{R}{L}, \quad Y = \sqrt{\frac{C}{L}} \quad (14)$$

I_0 and I_1 are the modified Bessel functions of zeroth and first order, and δ_t and u_t are the unit delta and unit step functions.

5 Multiconductor Lossy Lines

A lossy multiconductor system of n wires is considered. If a TEM (transverse electromagnetic) mode of wave propagation is assumed, the Telegrapher Equations[10, 11]

$$\frac{\partial}{\partial x}\begin{bmatrix}\mathbf{v}\\ \mathbf{i}\end{bmatrix} = -\begin{bmatrix}\mathbf{0} & \mathbf{L}\\ \mathbf{C} & \mathbf{0}\end{bmatrix}\frac{\partial}{\partial t}\begin{bmatrix}\mathbf{v}\\ \mathbf{i}\end{bmatrix} + \begin{bmatrix}\mathbf{0} & \mathbf{R}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{v}\\ \mathbf{i}\end{bmatrix} \quad (15)$$

describe the voltages $\mathbf{v}$ and currents $\mathbf{i}$ along the line. x and t denote distance along the line and time, respectively. $\mathbf{L}$ and $\mathbf{C}$ are the symmetric inductance and capacitance matrices of the multiconductor system; $\mathbf{R}$ is a *diagonal* matrix representing loss in the wires. The Laplace transform of Equation 15 is taken with respect to time and the following equation

$$\frac{\partial}{\partial x}\begin{bmatrix} \mathbf{v}(x,s) \\ \mathbf{i}(x,s) \end{bmatrix} = -\begin{bmatrix} \mathbf{0} & s\mathbf{L}+\mathbf{R} \\ s\mathbf{C} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{v}(x,s) \\ \mathbf{i}(x,s) \end{bmatrix} \quad (16)$$

where $\mathbf{v}$ and $\mathbf{i}$ are functions of x and s, the Laplace variable, is obtained.

A change of basis is needed in order to uncouple variables. Define:

$$\mathbf{v}(x,s) = \mathbf{M_V}\mathbf{v_d}(x,s), \quad \mathbf{i}(x,s) = \mathbf{M_I}\mathbf{i_d}(x,s) \quad (17)$$

where $\mathbf{M_V}$ and $\mathbf{M_I}$ are invertible matrices.

This results in the equation

$$\frac{\partial}{\partial x}\begin{bmatrix} \mathbf{v_d}(x,s) \\ \mathbf{i_d}(x,s) \end{bmatrix} = -\begin{bmatrix} \mathbf{0} & \mathbf{Z} \\ \mathbf{Y} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{v_d}(x,s) \\ \mathbf{i_d}(x,s) \end{bmatrix} \quad (18)$$

where

$$\mathbf{Z} = \mathbf{M_V}^{-1}(s\mathbf{L}+\mathbf{R})\mathbf{M_I}, \quad \mathbf{Y} = \mathbf{M_I}^{-1}(s\mathbf{C})\mathbf{M_V} \quad (19)$$

In order that the basis transformation (Equations 17) uncouple the system, it is necessary to find $\mathbf{M_V}$ and $\mathbf{M_I}$ so that $\mathbf{Z}$ and $\mathbf{Y}$ are diagonal matrices. For arbitrary multiconductor systems, the members of the diagonal matrices $\mathbf{Z}$ and $\mathbf{Y}$ may be nonlinear functions of the Laplace variable s. In order that the uncoupled problem consist of a set of n single-conductor lossy lines, it is necessary for the matrices $\mathbf{Z}$ and $\mathbf{Y}$ to be linear in s, i.e., of the form:

$$\mathbf{Z} = s\mathbf{L}' + \mathbf{R}', \quad \mathbf{Y} = s\mathbf{C}' \quad (20)$$

where $\mathbf{L}'$, $\mathbf{R}'$, and $\mathbf{C}'$ are *diagonal numeric* matrices, i.e., they are independent of s.

Romeo and Santomauro [11] have shown that there exists a special case of coupled *lossless* lines for which $\mathbf{M_V} = \mathbf{M_I} = \mathrm{M}$ and $\mathbf{Z}$ and $\mathbf{Y}$ can be expressed explicitly in terms of the entries of $\mathbf{L}$ and $\mathbf{C}$. Extending their approach, the following assumptions are made:

Assumption 5.1 *The capacitance and inductance matrices* **L** *and* **C** *are* tridiagonal, *equivalent to restricting coupling to between adjacent wires only.*

Assumption 5.2 *The lines are identical, equally spaced, and edge-effects are negligible. This is equivalent to saying that the self-capacitance, self-inductance, series resistance, parallel conductance, capacitive and inductive coupling are the same for all wires of the line.*

It can be shown [8] that if the preceding two assumptions hold, a numeric matrix $\mathrm{M} = \mathbf{M_I} = \mathbf{M_V}$ exists such that $\mathbf{Y}$ and $\mathbf{Z}$ assume the form in Equations 20. M is independent of the entries of $\mathbf{R}$, $\mathbf{L}$ and $\mathbf{C}$, depending only on the number of wires n. Moreover $\mathbf{R}' = \mathbf{R}$, and the diagonal elements $\mathbf{L}'_{ii}$ and $\mathbf{C}'_{ii}$ of $\mathbf{L}'$ and $\mathbf{C}'$ are given by:

$$\mathbf{L}'_{ii} = \mathbf{L}_s + \mu_i\mathbf{L}_m, \quad \mathbf{C}'_{ii} = \mathbf{C}_s + \mu_i\mathbf{C}_m, \quad i = 1,\cdots,n \quad (21)$$

$$\mu_i = -2\cos\frac{i\pi}{n+1}, \quad i = 1,\cdots,n. \quad (22)$$

where $\mathbf{L}'_s$ ($\mathbf{C}'_s$) and $\mathbf{L}'_m$ ($\mathbf{C}'_m$) are the diagonal and off-diagonal elements of $\mathbf{L}$ ($\mathbf{C}$) respectively. M is given by:

$$\mathbf{M}_{ij} = \frac{\phi_{i-1}(\mu_j)}{\gamma_j}, \quad i,j = 1,\cdots,n \quad (23)$$

$$\gamma_j^2 = \sum_{i=1}^{n}(\phi_{i-1}(\mu_j))^2 \quad (24)$$

$$\phi_k(x) = x\phi_{k-1}(x) - \phi_{k-2}(x), \ \phi_0(x) = 1, \ \phi_1(x) = x \quad (25)$$

Each diagonal entry of $\mathbf{L}$, $\mathbf{R}$ and $\mathbf{C}$ represents an uncoupled simple lossy line, and the matrix $\mathbf{M}$ represents a linear memoryless $2n$-port network (see Fig. 4). *A key difference between lossless and lossy multiconductor lines is that whereas the lossless case can always be decomposed into uncoupled lossless lines (even when assumptions 5.1 and 5.2 above do not hold), lossy multiconductor lines cannot in general be decomposed into uniform constant-parameter uncoupled simple lossy lines.*

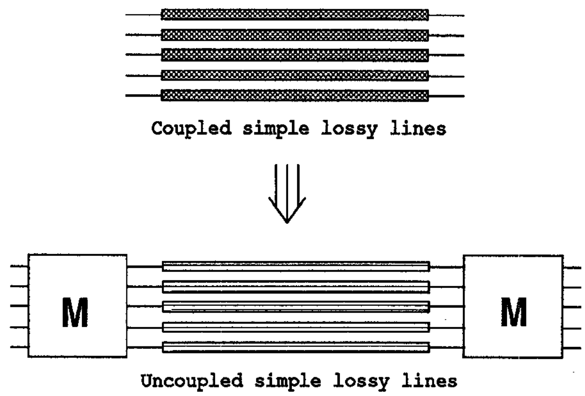

Figure 4: Multiconductor Decomposition

6 Experimental Results

The techniques described in the preceding sections were implemented in the circuit simulator SPICE3 version 3C.1 [25]. Waveform and computation speed comparisons with the segmentation methods are presented here for six circuits.

The circuits use digital BJT drivers with output rise-times of 500ps–2ns, connected by lossy interconnect to diode receivers or other BJT drivers. **raytheon1** has one fan-out, **raytheon2** has branching interconnect to three fan-outs, and **raytheon3** has a 2-wire multiconductor line; all three use identical interconnect parameters[3]. Interconnect parameters for **honeywell** and **mosaic** were taken from [2], and those for **mcnc** from [7]; these three circuits have one fan-out each. The number of segments used for the segmentation methods was kept small while ensuring reasonable accuracy. Table 1 compares execution times, and Figs. 5 – 7 are sample output waveforms. Figure 8 shows the variation of execution time vs. interconnect length for the three methods.

[3]These circuits were provided by Raytheon Co.

Circuit	Line Parameters		Execution Time[a]		
	R (ohms)	Z_0 (ohms)	lumped-RLC	pseudo-lumped	Convolution
raytheon1	0.2 /inch	50	1366.07s	526.41s	43.62s
raytheon2	0.2 /inch	50	691.53s	744.05s	65.38s
raytheon3	0.2 /inch	50	1158.96s	-	104.14s
mosaic	12.45 /cm	137	50.52s	47.82s	1.19s
honeywell	0.67 /cm	30	49.7s	42.76s	1.12s
mcnc	5.24 /cm	294	62s	337s	11s

[a]CPU times on a DEC 5400 running Ultrix 4.0

Table 1: Comparison of Execution Times

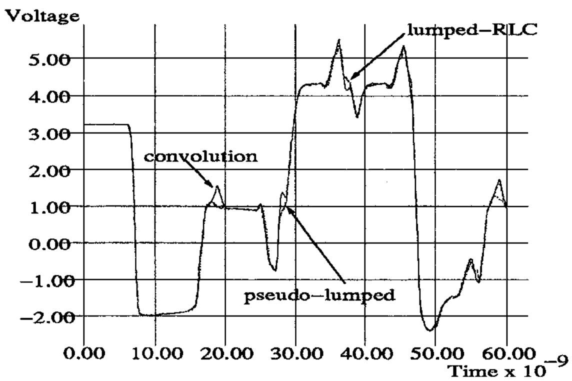

Figure 5: **raytheon1** receiver-end voltage

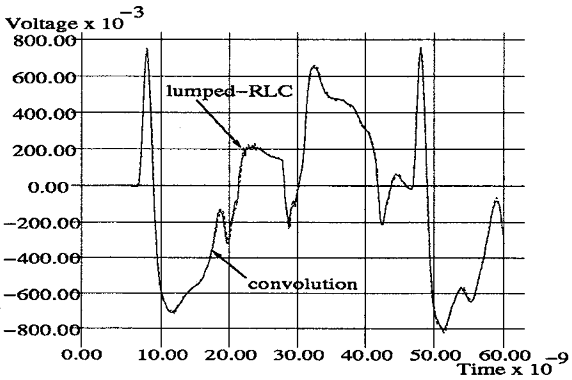

Figure 6: **raytheon3** far-end crosstalk

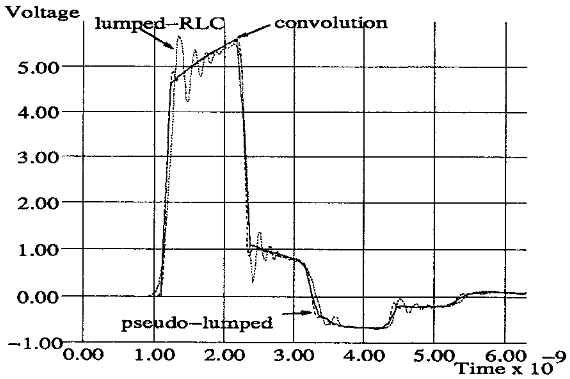

Figure 7: **mosaic** receiver-end voltage

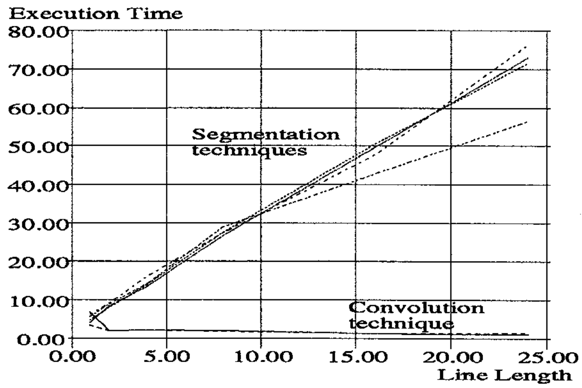

Figure 8: **mosaic**, **honeywell**: execution-time vs line-length

7 Conclusion

It can be seen from Table 1 and Figs. 5 – 8 that the enhanced convolution method performs favourably compared to the lumped-RLC and pseudo-lumped methods. That accuracy of results is maintained for all R, L and C is a direct consequence of using the improved convolution technique and explicit impulse-response formulae. For long lines, order-of-magnitude speedups are observed for some circuits. As line lengths become shorter, the speedups reduce because fewer segments are used by the segmentation techniques. An undesirable feature of the convolution approach is its quadratic time-complexity, i.e., the computation time required for simulation upto time T is proportional to approximately T^2, or more exactly, the square of the number of time-points during the simulation. This leads to disproportionately long simulation times when small timesteps are used - as seen in Fig. 8 for short lines (small line-delays force small timesteps). We are currently investigating approaches to reduce the time-complexity to approximately linear.

In conclusion, the convolution approach has been enhanced by using analytical formulae for simple lossy lines, along with accurate numerical convolution formulae. Simulation of a special case of lossy n-wire multiconductor lines has been shown to be equivalent to simulating n uncoupled simple lossy lines. Results of the technique on practical circuits with large nonlinearities show that it can be an order-of-magnitude faster than segmentation techniques for equivalent or better accuracy.

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