Computation of circuit waveform envelopes using an efficient, matrix-decomposed harmonic balance algorithm

P. Feldmann
Bell Laboratories
Murray Hill, NJ 07974

J. Roychowdhury
Bell Laboratories
Murray Hill, NJ 07974

Abstract

In this paper we introduce a novel algorithm for numerically computing the "slow" dynamics (envelope) of circuits in which a "fast" varying carrier signal is also present. The algorithm proceeds at the rate of the slow behavior and its computational cost is fairly insensitive to the rate of the fast signals. The envelope computation problem is formulated as a differential-algebraic system of equations (DAEs) in terms of frequency-domain quantities (e.g. amplitudes and phases) that capture the fast varying behavior of the circuit. The solution of this DAE represents the "slow" variation of these quantities, i.e., the envelope. The efficiency of this method is the result of using the most appropriate method for each of the circuit modes: harmonic balance for the fast behavior and time-domain integration of DAEs for the slow behavior. The paper describes the theoretical foundations of the algorithm and presents several circuit analysis examples.

1 Introduction

In many electronic circuits, and in particular those used for RF communication applications, a high-frequency signal is used as carrier for the information signal that varies at rates that are several orders of magnitude slower. Examples of such circuits are automatic gain-control circuits, phase-lock loops, DC-DC converters, switched capacitor circuits etc..

Traditional time-domain circuit simulation methods rely on numerical integration of differential-algebraic equations (DAEs) and are extremely inefficient when applied to such circuits. The time-domain integration algorithm is forced to follow the "fast" varying signal for a length of time sufficient to study the behavior of the "slow" information signal. It is easy to see that the number of time-points that the algorithm needs to compute becomes prohibitive.

The steady state of such circuits is more naturally analyzed in the frequency domain using a method such as the harmonic balance [1, 2, 3]. Harmonic balance analysis has recently become practical for large circuits subjected to multi-tone excitations [4, 5, 6] thanks to the use of Krylov subspace based methods [9] for solving the large and dense linear systems associated with harmonic balance. Unfortunately steady-state computation is not always sufficient. Often, designers are interested to study the response of their system to transient phenomena that typically occur at the rate of the "slow" information signal. Such phenomena are naturally analyzed in the time domain, and, ideally, the analysis should not have to follow the fast rate of the carrier signal.

In this paper, we introduce a novel algorithm for the direct computation of the "slow" information signal, the *envelope*. This algorithm is significantly more efficient than plain time-domain methods and achieves the same level of accuracy. The algorithm proceeds at the rate of the slow phenomena and its computational cost is fairly insensitive to the rate of the fast signals. The envelope computation problem is formulated as DAEs in terms of the frequency-domain quantities (e.g. amplitudes and phases) that capture the fast varying behavior of the circuit. The solution of this DAE represents the "slow" variation of these quantities, i.e., the envelope. The efficiency of this method results from using the most appropriate method for each of the modes of the circuit: efficient harmonic balance with the decomposed matrix algorithm [4, 5] for the fast behavior and time-domain integration of DAEs for the slow behavior.

The matrix-decomposed harmonic balance algorithm uses a Krylov subspace based iterative method to solve the large and dense linear systems that arise in the innermost loop of the harmonic balance algorithm. Krylov methods derive all the information about the linear system matrix from products of the matrix (and/or its transpose) with arbitrary vectors. In other words, unlike sparse direct methods, the matrix does not need to be stored and operated upon explicitly. Instead, Krylov methods only require procedures that compute products of the matrix (or its transpose) with arbitrary vectors. This property makes a tremendous difference in the case of the harmonic balance algorithm. Here, the linear system matrix is in fact the harmonic balance Jacobian matrix which is very large and contains dense sub-blocks. On the other hand, the multiplication of the Jacobian matrix or its transpose with a vector can be decomposed into a sequence of linear operations that can be executed in nearly linear time and storage through the use of fast algorithms (such as the FFT). The complete Jacobian matrix is never computed or stored explicitly. The use of Krylov subspace based iterative methods, in conjunction with powerful preconditioners [4] permit the harmonic balance algorithm to be applied to circuits consisting of thousands of non-linear components.

The envelope computation algorithm formulates the circuit analysis problem as a system of nonlinear DAEs in terms of the time-varying amplitudes and phases of the "fast" signal oscillations. The DAE solver requires in its inner loop the solution of a linear system with a structure similar to the harmonic-balance balance inner problem. The system is solved with a preconditioned QMR algorithm [9] (a Krylov subspace method). As a consequence, the cost of this analysis is practically linear in the size of the circuit and can be applied to very large problems.

Recently, Hewlett-Packard announced the commercial availability of an envelope simulator [7]. This product is based on similar principles as those discussed in this paper, but being based on the traditional harmonic balance algorithm [1] it is limited to much smaller circuits.

The remainder of this paper is organized as follows: First, we derive the formulation of the mixed HB-DAE equations. Then, we show that the solution algorithm is a standard implicit DAE solver and the nonlinear system that must be solved in its inner loop is a modification of the harmonic balance problem. We describe the solution of this inner system by means of a the matrix-decomposed Krylov method. The last section of the paper describes the analysis of several circuit examples.

2 HB-DAE system formulation

We start from the original circuit equations:

$$\frac{d}{dt}q(x(t)) + f(x(t)) = b(t). \tag{1}$$

We are seeking a solution of the form

$$x(t) = X^T(t) \cdot w(t). \tag{2}$$

where

$$w(t) = [\,1 \quad 2\cos\omega t \quad -2\sin\omega t \quad \dots \quad -2\sin M\omega t\,]^T$$

is a properly truncated Fourier basis and $X(t)$, the *envelope*, is a vector of "slowly" varying Fourier coefficients. The notion of "slow" will be clarified in the sequel. We further assume that all the components of the circuit equation (1) have the same form

$$\begin{aligned} q(t) &= Q(X(t))^T \cdot w(t), \\ f(t) &= F(X(t))^T \cdot w(t), \\ b(t) &= B(X(t))^T \cdot w(t), \end{aligned} \tag{3}$$

where $Q(X(t))$, $F(X(t))$ and $B(X(t))$ are also vectors of "slowly" varying Fourier coefficients. Under these assumptions, from (1), we obtain the following system of *functional* circuit equations

$$\frac{d}{dt}Q(X(t))^T w + Q(X(t))^T \underbrace{\frac{d}{dt}w}_{\Omega w} + F(X(t))^T w = B^T w. \tag{4}$$

where the differentiation of the Fourier basis is accomplished through multiplication with the matrix

$$\Omega = \begin{bmatrix} 0 & & & & & \\ & 0 & -\omega & & & \\ & \omega & 0 & & & \\ & & & \ddots & & \\ & & & & 0 & -M\omega \\ & & & & M\omega & 0 \end{bmatrix} \tag{5}$$

By assuming that the envelope signals and their derivatives are band-limited it can be shown that (4) can be discretized (through a Galerkin procedure) to yield the following system of differential equations

$$\begin{aligned} \frac{d}{dt}Q_1(X(t)) + Q_1(X(t)) + F_1(X(t)) &= B_1(t) \\ \frac{d}{dt}Q_2(X(t)) - \omega Q_3(X(t)) + F_2(X(t)) &= B_2(t) \\ \frac{d}{dt}Q_3(X(t)) + \omega Q_2(X(t)) + F_3(X(t)) &= B_3(t) \\ &\dots \end{aligned} \tag{6}$$

or, in matrix notation,

$$\frac{d}{dt}Q(X(t)) + \Omega^T Q(X(t)) + F(X(t)) = B(t). \tag{7}$$

This system is a DAE and can be solved using a standard integration algorithm. Numerical DAE integration algorithms find solutions at a sequence of discrete time=points, $t_i, i = 0, 1, \dots, n$. Using a linear multistep formula (the method of choice for the numerical solution of DAEs) leads at each time-point t_i to the approximation

$$\frac{d}{dt}Q(X_i) \approx \alpha_i Q(X_i) + \beta_i. \tag{8}$$

Here we used the short-hand notation $X_i = X(t_i)$. Applied to equation (7), yields

$$F(X_i) + \alpha_i Q(X_i) + \beta_i + \Omega^T Q(X_i) = B(t_i). \tag{9}$$

The computation of $F(X_i)$ and $Q(X_i)$ is done in the following way: Given X_i which is a set of Fourier coefficients by applying an inverse DFT we obtain the time domain vector $x_i = [x_i(\tau_1) \ \ x_i(\tau_2) \ \ \ldots \ \ x_i(\tau_N)]^T$. Note that the time points $\tau_1, \tau_2, \ldots, \tau_N$ are spaced according to the "fast" mode of the circuit, as opposed with the time points t_i spaced according to the "slow" mode. We evaluate the nonlinear functions $q(\cdot)$ and $f(\cdot)$ on a time-point by time-point basis and obtain the time-domain vectors $f = [f(\tau_1) \ \ f(\tau_2) \ \ \ldots \ \ f(\tau_N)]^T$ and $q = [q(\tau_1) \ \ q(\tau_2) \ \ \ldots \ \ q(\tau_N)]^T$. Finally we apply the DFT to these vectors and obtain the Fourier coefficients F and Q. In matrix notation

$$F = \Gamma^{-1} \begin{bmatrix} f(\cdot) & & \\ & \ddots & \\ & & f(\cdot) \end{bmatrix} \Gamma X_i = \Gamma^{-1}\mathcal{F}(\Gamma X_i). \tag{10}$$

This procedure relies on the "slow" variation of the Fourier coefficients $X(t)$ and assumes that they are practically constant over a period of time $T = 2\pi/\omega$. As a result, for any given time point t_i, $F(t_i)$ and $Q(t_i)$ can be computed only from $X(t_i)$ using the DFT, IDFT pair as in the case of regular harmonic balance analysis. Equation (9) becomes

$$\Gamma^{-1} \underbrace{(\mathcal{F} + \alpha_i \mathcal{Q})}_{\mathcal{G}_i}(\Gamma X_i) + \Omega^T \Gamma^{-1} \mathcal{Q}(\Gamma X_i) = B(t_i) - \beta_i. \tag{11}$$

This is an algebraic system of equations with the same structure as a regular harmonic balance system. The only modification is the replacement of the nonlinear operator $\mathcal{F}$ by

$$\mathcal{G}_i = \begin{bmatrix} f(\cdot) + \alpha_i q(\cdot) & & \\ & \ddots & \\ & & f(\cdot) + \alpha_i q(\cdot) \end{bmatrix} \tag{12}$$

and the modification of the right-hand-side by the vector β_i.

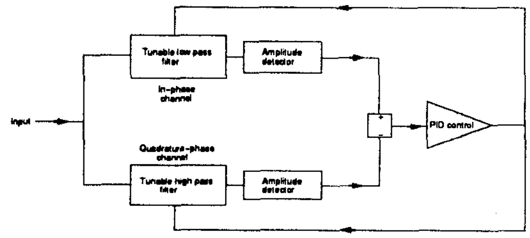

Figure 1: Tunable filter circuit

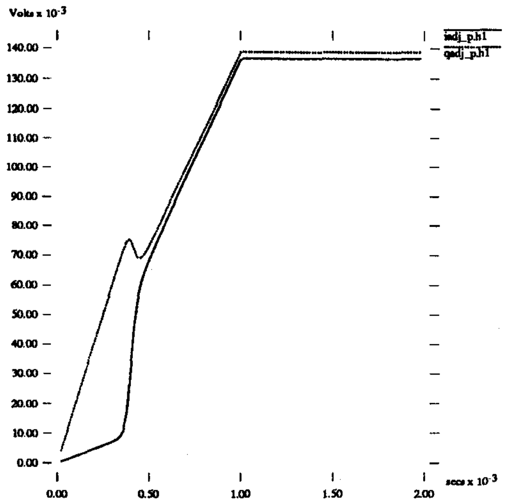

Figure 2: Amplitude of filter output first harmonic

3 Experimental Results

The envelope computation technique of the preceding sections has been prototyped in an in-house circuit simulator. Results on two circuits are presented in this section.

The first circuit, depicted in Figure 1, consists of two tunable filters excited by a common sinusoidal source, and a control loop. The two filters are not identical – one is high-pass while the other is low-pass. The requirement is that the magnitudes of the two filter output signals be equal. The magnitudes of the filter outputs are compared and the difference is fed back through a control loop to tune the filters, as shown. The purpose of the loop is to shift the filters' transfer functions so that they both have the same magnitude gain at the frequency of the input signal. An envelope simulation was carried out to detect possible instabilities or ringing in the feedback control mechanism. In this analysis, the amplitude of the sinusoidal input signal is ramped linearly from zero to its full value.

The envelopes of the first harmonic components of the filter outputs are shown in Figure 2. The initial

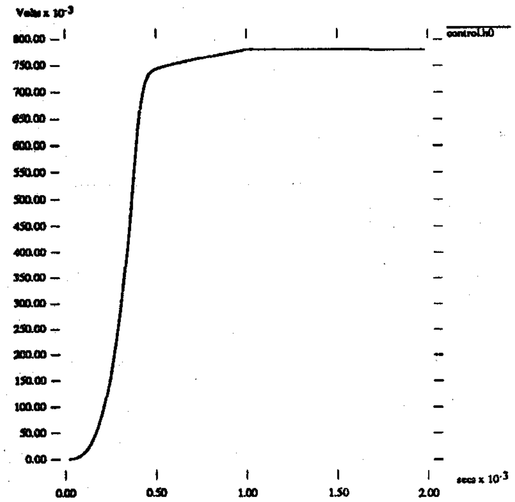

Figure 3: DC component of feedback control voltage

difference in the two climb rates indicates that the filters are mismatched with respect to the frequency of excitation. After a delay, however, the control mechanism forces the amplitudes to track each other. The reason for the delay is found in Figure 3, which depicts the time-varying DC component of the control signal. The control loop operates by switching on a device at around 700mV; before this value is reached, the feedback is not active and the filter outputs do not track each other. The simulation of this circuit took about 25 minutes on a SparcStation 20.

The second circuit, an automatic gain control loop, is shown in Figure 4. The circuit uses feedback control to stabilize the amplitude of the signal at the output of the variable gain amplifier. In the simulation, the input amplitude is ramped from zero to its full value, as shown in Figure 5. The envelope of the waveform at the output of the variable gain amplifier is shown in Figure 6. It can be seen that, up to a point, this amplitude rises at a faster rate than the input, indicating that the control loop has set the gain high until the reference level is reached. At this point, the gain is reduced in synchrony with the increasing level of the input. A slight overshoot can be seen. The feedback voltage controlling the amplifier gain is shown in Figure 7.

Figure 8 depicts the envelope of the amplified signal when the feedback loop response is made slower by increasing the time-constant of the amplitude detector. It can be seen that the control process verges on instability and displays oscillations. This occurs because the control voltage (Figure 9), after it starts

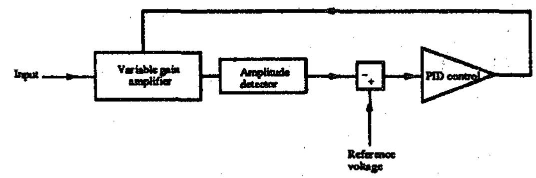

Figure 4: Automatic gain control circuit

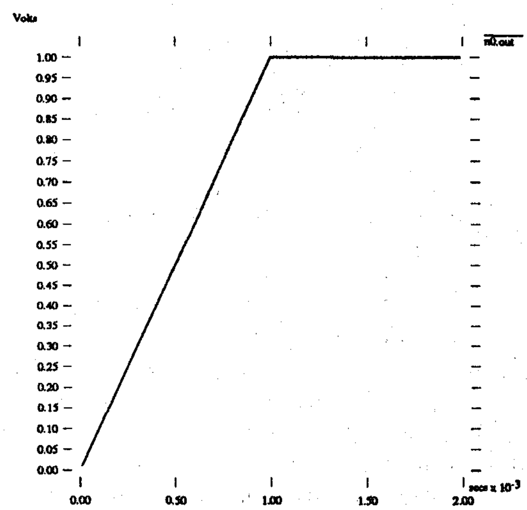

Figure 5: Ramped amplitude of AGC input

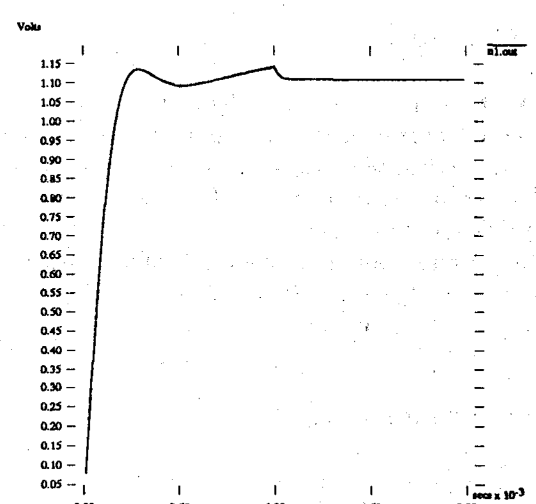

Figure 6: Amplitude of first harmonic at amplifier output

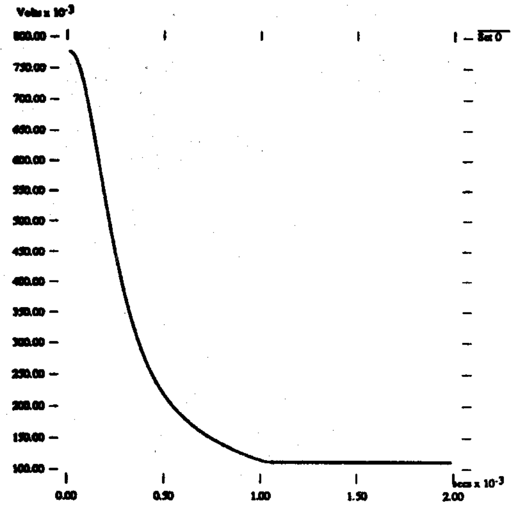

Figure 7: DC component of control voltage

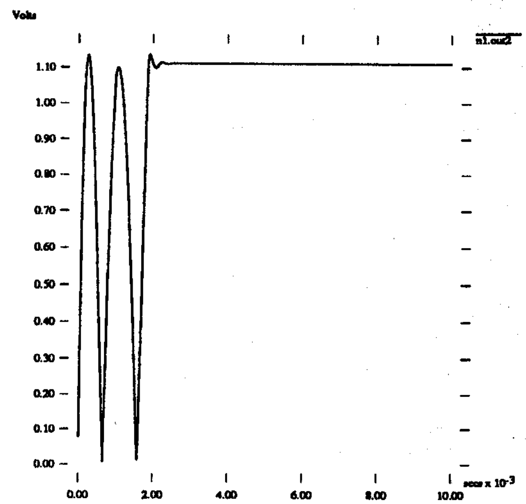

Figure 8: Underdamped output envelope

reducing the gain, does not readjust fast enough to the fact that the gain has reduced sufficiently to meet the target output amplitude. Even though the system is a driven one (at the frequency of the input signal), the control loop generates autonomous behavior in the envelope at a much lower rate than the signal itself. The stability of such control loops is frequently of concern to designers. Apart from brute force transient simulation which is usually impractically long, we are not aware of any means other than envelope simulation by which such concerns can be addressed.

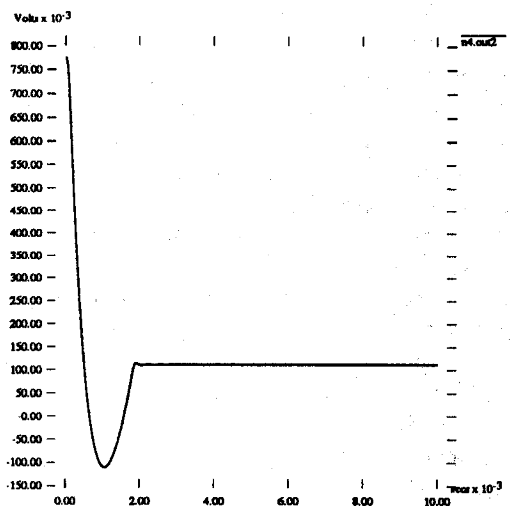

Figure 9: Underdamped DC component of control

4 Conclusion

We have presented an algorithm that combines the best features of the time-domain and frequency-domain methods for studying systems that operate at widely separated frequencies. This method is particularly suitable to problems where the slow mode of operation is autonomous or is a result of a transient excitation. The algorithm is an extension of the factored-matrix harmonic balance algorithm and therefore can be applied efficiently to very large circuits.

Acknowledgments

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