Analyzing Strongly Nonlinear Multitone Circuits by Multi-Time Methods

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Abstract

Harmonic balance and traditional time-domain simulation techniques are limited in their applicability when strong nonlinearities and multi-tone signals are both present. In this paper, new simulation algorithms are described and used to analyse strongly nonlinear microwave circuits with multi-tone excitations. The techniques are based on representing multi-tone waveforms as functions of *several* "time" variables, and solving a partial differential form of the circuit equations using time-domain methods. The methods scale linearly with increasing circuit size, regardless of the number of nonlinearities. A switching mixer is analyzed using the new methods. Speedups of over two orders of magnitude over traditional techniques are obtained. Moreover, the multi-time representations obtained from the new methods provide intuitive and powerful visualisations of the operation of the circuit.

1 Introduction

Microwave circuits typically operate in mildly nonlinear regimes. This fact is exploited for their simulation - for example, harmonic balance (e.g., [RN88, KWSV90, GS91, dCP97]) relies on the absence of strongly nonlinear behaviour to be effective. In recent years, however, IC techniques have been applied to microwave design, resulting in an increase in the nonlinearity encountered. For example, circuits like switching mixers depend inherently on strongly nonlinear action for their operation. For one tone circuits, time-domain techniques like shooting suffice to handle the nonlinearity, but when there are several tones, both harmonic balance and time-domain methods break down or become very inefficient.

Recently, we introduced novel methods for solving strongly nonlinear multi-tone systems, based on the *multi-rate partial differential equation* or *MPDE* [BWLBG96, NL96, Roy98, Roy97]. By representing signals using more than one time variable (see later for an illustration), effective analysis of strong nonlinearities in the presence of widely-separated time scales becomes possible. The methods are efficient for large circuits with many nonlinearities, important in integrated RF/microwave applications. In this paper, we describe the MPDE-based techniques and apply them to switching mixers for microwave and wireless applications. The new methods can be more than two orders of magnitude faster than traditional techniques.

In Section 2, previous simulation techniques relevant to the multirate problem are reviewed briefly. The MPDE formulation, together with new numerical methods based on it, is described in Section 3. Application of the new methods to is described in Section 4.

2 Previous methods and limitations

Existing time-domain methods such as transient integration and shooting, while suitable for strong nonlinearities, have difficulty with widely separated time-scales. They are forced to follow the details of waveforms at the fastest time scale, for the much larger durations of the slower time scales. This results in an excessive number of simulation timepoints and also accuracy loss.

Frequency-domain methods like harmonic balance (e.g., [KWSV90]) can, on the other hand, handle several time scales or tones, but have difficulty with strong nonlinearities. The reason is that strong nonlinearities tend to generate signals with sharp edges and corners, which are inefficient to represent in the frequency domain because they require many Fourier components. Strong nonlinearities also cause problems with preconditioning, needed in recent harmonic balance methods (e.g., [MFR95]).

Mixed frequency-time methods, such as those of Ushida and Chua [UC84] and Kundert et al [KWSV90], have attempted to combine the advantages of both domains. However, these techiques rely on highly localized sampling, resulting in numerical ill-conditioning. They are also limited to only one strongly nonlinear tone. The specialized program SWITCAP [FTW83], designed for switched-capacitor circuits, uses idealized switch models to achieve significant speedups over more general algorithms. This approach ignores important nonlinear effects; in particular, it is limited in its capability to predict signal path harmonic distortion, a critical figure of merit for SC designs.

Multiple time concepts have appeared previously in several places. To our knowledge, the first use of multivariate functions and related PDE forms was in asymptotic expansion analysis [KC81] of nonlinearly perturbed simple harmonic oscillators. More recently, Ngoya and Larchevèque [NL96] used multiple time variables to obtain an envelope simulation procedure. Brachtendorf [BWLBG96] used the PDE form to obtain a simple and elegant derivation of multi-tone harmonic balance.

3 MPDE-based numerical algorithms

In this section, an overview of the MPDE and related numerical techniques is presented. Many details are omitted and may be found in [Roy98].

The key to the MPDE formulation is the use of multivariate functions (functions of several time variables) to represent signals with separated time scales efficiently. To understand the concept, consider the product of a 1 Hz sine wave and a 1Ghz pulse train, given by:

$$y(t) = \sin(2\pi t)\,\text{pulse}\left(\frac{t}{10^9}\right) \tag{1}$$

Figure 1 depicts $y(t)$, with the pulse period of 10^9 changed to 50 for viewing convenience. This quasi-periodic signal is expensive to represent in the time domain because 10^9 pulses of different shapes need to be sampled before the waveform repeats. It is this problem that makes traditional time-domain techniques like SPICE's transient analysis inefficient for such signals. Representation in the frequency domain as a two-tone signal is also inefficient because the pulses require many Fourier components for accuracy.

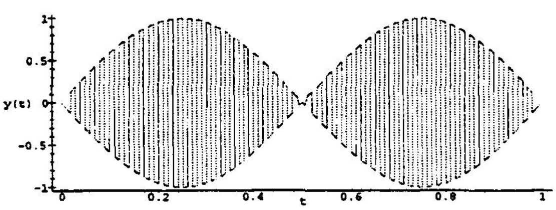

Figure 1: $y(t)$

Consider, however, the function of two variables obtained by replacing the 'slow' time component by t_1 and the 'fast' time component by t_2:

$$\hat{y}(t_1,t_2) = \sin(2\pi t_1)\,\text{pulse}\left(\frac{t_2}{10^9}\right) \tag{2}$$

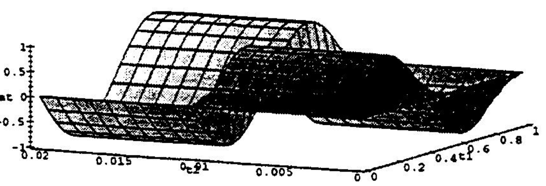

Figure 2: $\hat{y}(t_1,t_2)$

$\hat{y}(t_1,t_2)$, a *bi-variate form* of $y(t)$, is shown in Figure 2. Notice that it is easy to represent $\hat{y}$ accurately using relatively few numerical samples, in contrast to $y(t)$ in Figure 1. The number of samples does not depend on the separation of the two time scales, which merely determines the scaling of the axes. Moreover, $y(t)$ can be easily obtained by interpolation from samples of $\hat{y}(t_1,t_2)$, using the fact that $y(t) = \hat{y}(t,t)$ and that $\hat{y}(t_1,t_2)$ is periodic in each argument.

This observation is the basis of the MPDE formulation, in which all the waveforms in a circuit are represented in their bi-variate forms (or multivariate forms if there are more than two time scales). The key to efficiency is to solve for these waveforms directly, without involving the numerically inefficient one-dimensional forms at any point. To do this, it is necessary to first describe the circuit's equations using the multivariate functions. The traditional form of a circuit's equations, used in all simulators, is the Differential-Algebraic Equation (DAE):

$$\dot{q}(x) + f(x) = b(t) \qquad (3)$$

$x(t)$ is the vector of circuit unknowns (node voltages and branch currents); q denotes the charge/flux terms and f the resistive terms; $b(t)$ is the vector of excitations to the circuit (typically from independent voltage/current sources). It can be shown [Roy98] that if $\hat{x}(t_1,t_2)$ and $\hat{b}(t_1,t_2)$ denote the bi-variate forms of the circuit unknowns and excitations, then the following MPDE is the correct generalization of (3) to the bi-variate case:

$$\boxed{\frac{\partial q(\hat{x})}{\partial t_1} + \frac{\partial q(\hat{x})}{\partial t_2} + f(\hat{x}) = \hat{b}(t_1,t_2)} \qquad (4)$$

More precisely, if $\hat{b}$ is chosen to satisfy $b(t) = \hat{b}(t,t)$, and $\hat{x}$ satisfies (4), then it can be shown that $x(t) = \hat{x}(t,t)$ satisfies (3). Also, if (3) has a quasi-periodic solution, then (4) can be shown to have a corresponding bi-variate solution.

By solving the MPDE numerically in the time domain, strong nonlinearities can be handled efficiently. The following new methods have been developed for solving (4):

1. <u>Quasi-periodic time-domain methods (MFDTD and HS):</u> Quasi-periodic solutions are found by enforcing bi-periodic boundary conditions on the MPDE. In the Multivariate Finite Difference Time Domain (MFDTD), a (4) is discretized on a grid in the t_1-t_2 plane by approximating the differentiation operators with a numerical differentiation formula. The resultant system of nonlinear equations, together with the bi-periodic boundary conditions, is solved using a nonlinear solution method. The grid is refined adaptively so that the solution is captured efficiently. Another purely time-domain method, Hierarchical Shooting (HS), is a generalization of the traditional shooting method to multiple time scales. Both MFDTD and HS are appropriate for circuits with no sinusoidal waveform components, such as power converters.
2. <u>Quasi-periodic mixed frequency/time method (MMFT):</u> In some circuits, the slow-scale signal path is often almost linear, while the fast-scale action is highly nonlinear. The linearity of the signal path can be exploited by expressing the slow scale components in a short Fourier series, and solving the mixed frequency/time system of equations. This Multivariate Mixed Frequency Time (MMFT) method is often more efficient for switched-capacitor filters and switching mixers.
3. <u>Time domain envelope methods (TD-ENV):</u> Envelope-type solutions can be generated from the MPDE by applying mixed initial/periodic boundary conditions. Novel time-domain methods based on FDTD or shooting along the fast time scale, and transient integration along the slow time scale, have been devised [Roy98]. These techniques are capable of handling circuits with nonlinearities on a fast time scale, e.g., power converters, switched-capacitor filters, switching mixers, etc..

The above numerical techniques generate sparse matrices with near diagonal or block-diagonal structure, which makes it convenient to use iterative linear solution methods (e.g., [Saa96, MFR95]) to solve large circuits efficiently.

4 Application to a switching mixer circuit

In this section, the MPDE-based numerical methods described above are applied to a double-balanced switching mixer and filter circuit for calculating intermodulation distortion. The RF input to the mixer was a 100kHz sinusoid with amplitude 100mV; this sent it into a mildly nonlinear regime. The LO input was a square wave of large amplitude (1V), which switched the mixer on and off at a fast rate (900Mhz).

The bi-variate forms of the RF and LO inputs are shown in Figs. 3 and 4 respectively. Note that the RF input varies only along the slow time axis, while the LO input does so only along the fast axis.

The mixer output, obtained using MFDTD, is shown in Figure 5. Observe that this signal is a true multi-tone signal, for there is variation along both axes. Note also the sinusoidal shape along the slow time scale, indicating that the RF input remains relatively undistorted along its time scale. Along the fast time scale, on the other hand, a charging/discharging behaviour can be seen – this is due to the effect of switching and subsequent filtering. The new methods are not only efficient and accurate, but the three-dimensional representation also provides a new and powerful visualisation of the operation of the circuit.

The harmonic distortion of the output could be obtained from Figure 5 by a Fourier analysis along the slow time scale. However, given that the signal path is expected to be relatively linear, this circuit is ideal for the MMFT method. For MMFT, 3 harmonics were taken in the RF tone $f_1 = 100$kHz (corresponding to the t_1 variable). The LO tone at $f_2 = 900$MHz was handled by time-domain shooting in the t_2 variable. The output of the MMFT algorithm is a set of time-varying harmonics that are periodic with period $T_2 = \frac{1}{f_2}$. The first harmonic is shown in Figure 6. This plot contains information about all mix components of the form $f_1 + if_2$, i.e., the frequencies 900.1 Mhz, 1800.1 Mhz, etc.. The main mix component of interest, 900.1 Mhz, is found by taking the fundamental component of the waveform in Figure 6. This has an amplitude of 60mV.

The third harmonic is shown in Figure 7. It contains information about the mixes $3f_1 + if_2$, i.e., the frequencies 900.3 Mhz, 1800.3 Mhz, etc.. The amplitude of the 900.3 Mhz component can be seen to be about 1.1mV; hence the distortion introduced by the mixer is about 35dB below the desired signal.

The circuit was also simulated by traditional (univariate) shooting for comparison. The output from univariate shooting is shown in Figure 8. This run, using 50 steps per fast period, took almost 300 times as long as the new algorithms.

5 Conclusion

We present efficient MPDE-based simulation algorithms for microwave circuits with several tones and strong nonlinearities. We apply the methods to a switching mixers, which is difficult to simulate by traditional methods. The new methods are much faster than previous ones (e.g., 300x speedup) and also more accurate. Further, they can provide results in three-dimensional form, which is a powerful new way of visualizing the operation of circuits with widely separated time scales.

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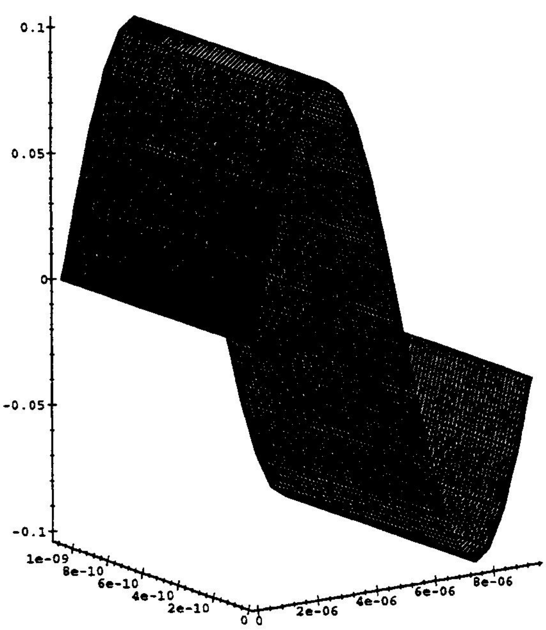

Figure 3: Switching mixer: RF input at 100kHz (bivariate form)

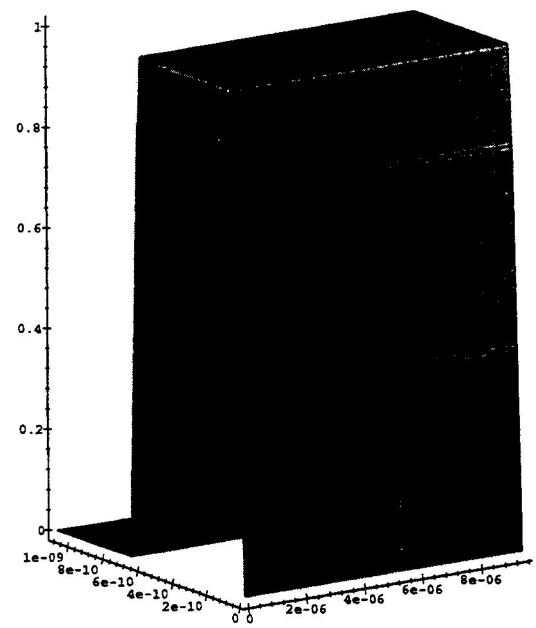

Figure 4: Switching mixer: LO input at 900MHz (bivariate form)

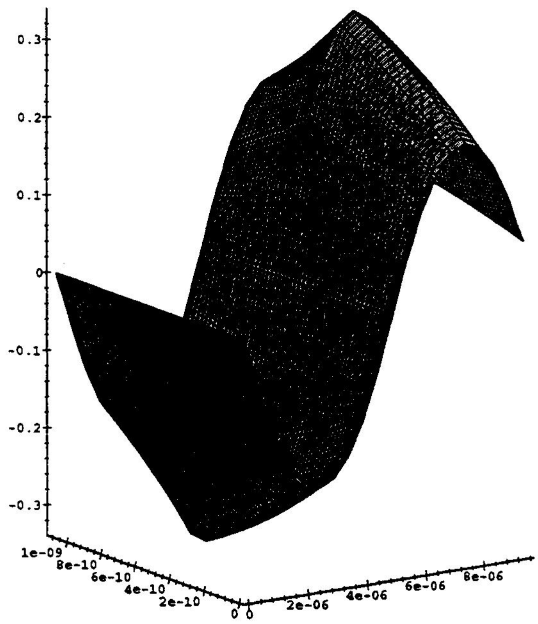

Figure 5: Mixer output (bivariate form) from MFDTD method

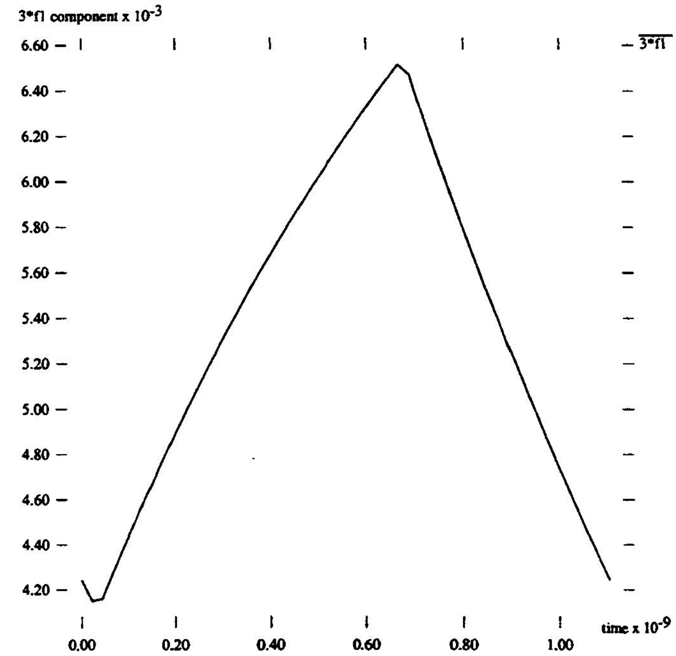

Figure 7: Third harmonic as a function of fast time (MMFT method)

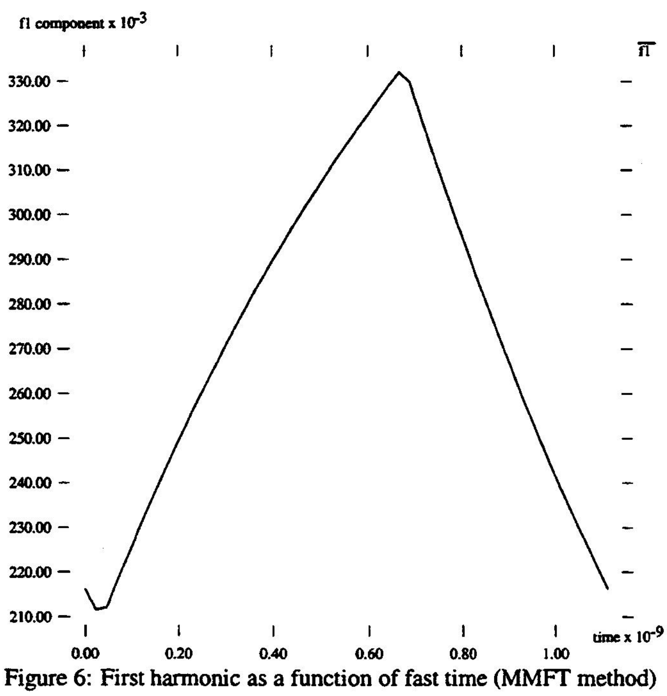

Figure 6: First harmonic as a function of fast time (MMFT method)

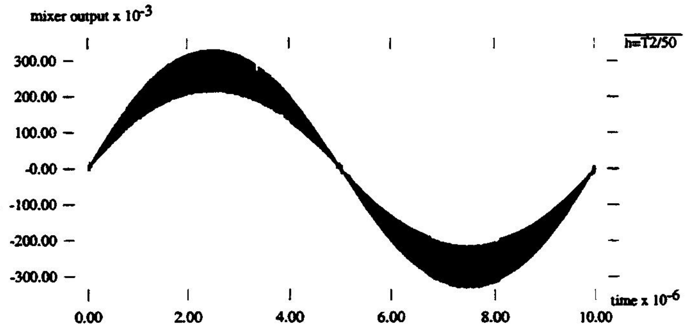

Figure 8: Mixer output from univariate shooting

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