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Analytical Equations For Predicting Injection Locking in LC and Ring Oscillators

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***Abstract*— In this paper, we present a analytical method for predicting injection locking in the LC and ring oscillators. Our method is very convenient for analyzing the injection locking phenomenon in oscillators, since it doesn't require the *Q* factor, as the Adler equation [1] does. We show that our analytical solution is equivalent with Adler's equation for LC oscillators. Moreover, our method can predict injection locking in ring oscillators, where Adler's equation fails since the operating mechanism is different. We demonstrate the proposed techniques on LC and ring oscillator circuits, comparing results from our analytical equations against full SPICE-level circuit simulations. Numerical experiments show our methods are able to predict injection locking in oscillators with great accuracy.**

I. Introduction

Injection locking [4] is a nonlinear phenomenon that can be observed in many natural oscillators: when an oscillator is perturbed by a weak signal whose frequency is close to the oscillator's free-running frequency, the oscillator's frequency changes to be identical to that of the perturbation signal. Recently, this phenomenon has been increasingly used in high speed oscillator designs. In optics, injection locking has been used in lasers to improve the frequency stability and reduce the frequency noise of laser diodes [5], [8]. In electronic system, injection locking is a well known and practical technique for phase locked loops (PLLs) to increase pull-in range and reduce output phase jitter [12]. As a result, fast and accurate prediction of injection locking is very important.

The direct computational method for analyzing injection locking is to simulate the oscillator circuit at the SPICE level, and to compare the circuit's response to the injected signal. Unfortunately, predicting injection locking via SPICE-like simulations can be extremely difficult: The accurate simulation of oscillators requires large number of small timesteps for each oscillation cycle, and a high *Q* oscillator may need thousands of cycles to lock to the external signal. In addition, for small injections at frequencies close to the oscillator's natural frequency, it is very difficult to observe injection locking from time domain waveforms, as the beat note – periodic variations of frequency and amplitude – due to the perturbation is very small. Since the direct simulation method is usually inefficient and inaccurate for predicting injection locking, a method to capture injection locking without performing full circuit simulation is of great interest.

To overcome the shortcoming of full circuit simulations, various analytical approaches [6], [9], [10] are used to predict injection locking in oscillators. They are all based on Adler's classic 1946 paper [1], which provides a simplified quantitive explanation of the phenomenon for simple harmonic oscillators, leading to formulae for their lock range. Adler's equation provides a simplified way to predict injection locking in LC oscillators. However, this method is not general, being derived form the concept of LC tank and limited to harmonic oscillators with an explicit *Q* factor. As a result, it cannot be applied to analyze other oscillators, such as ring oscillator, which is widely used in digital circuits for frequency synthesis, since the operating mechanism is totally different.

In [7], an efficient numerical approach that overcomes the limitation of the Adler equation is presented. The method exploits the perturbation projection vector (PPV) [2] of oscillators to predict injection locking. Developing upon a rigorous nonlinear phase macromodel for oscillators [2], this technique is applicable to any oscillators, regardless of operating mechanism. Moreover, the method improves significantly on Adler's method in terms of accuracy for LC oscillators, as it uses the exact expression of the circuit equations instead of the first-order approximation proposed by Adler. However, this method requires full circuit descriptions at SPICE-level, and the calculation of the PPV can be complex and difficult as the circuit size grows. Hence, this method can be inconvenient for fast prediction of injection locking in oscillators.

In this paper, we present analytical solutions for the fast prediction of injection locking in the LC and ring oscillators. We first derive analytical representations of the PPV of the LC and ring oscillators, based on some reasonable assumptions and simplifications. We then apply the analytical PPVs to the method in [7] and derive the analytical equations for predicting injection locking in the LC and ring oscillators. We show that our solution for LC oscillator is equivalent with Adler's equation, except our equation doesn't require the *Q* factor. Our method is more convenient than Adler's approach, since the *Q* factor is not easy to be calculated accurately, even for a simple LC oscillator circuit. Since Adler's equation cannot be applied to ring oscillators, we verify our analytical equation for ring oscillators using full SPICE-level circuit simulations. Numerical results show that our analytical equation is able to predict injection locking in ring oscillators with excellent accuracy. Our analytical solution for ring oscillator can be very useful for designers, since it can provide fast and accurate prediction of injection locking in ring oscillators where other injection locking approaches (*e.g.*, Adler's equation) fail.

The remainder of the paper is organized as follows. In Section II, we briefly review previous injection locking approaches. In Section III, we derive analytical equations for predicting injection locking in the LC and ring oscillators. In Section IV, we present simulation results on two oscillator examples and verify our analytical equations.

II. Previous Injection Locking Approaches

Various approaches have been proposed for prediction injection locking in oscillators. In this section, we briefly summarize two typical injection locking prediction schemes: one is the analytical method proposed by Adler (Adler's equation), the other is a semi-analytical method based on the PPV of oscillators.

A. Adler's Equation

In [1], Adler derived the phase dynamics of the oscillator from the phase and amplitude relationship between the oscillator and the injected signal. If an oscillator is perturbed, but not locked by the perturbation signal, we will observe a beat note. The following equation describes the instantaneous beat frequency of the LC oscillator perturbed by an external signal:

$$\frac{d\alpha}{dt} = -\frac{V_{inj}}{V_0}\frac{\omega_0}{2Q}\sin(\alpha) + \Delta\omega_0, \tag{1}$$

where V_{inj} is the injected voltage, V_0 and ω_0 are the output voltage and frequency of the unperturbed oscillator, and $\frac{d\alpha}{dt}$ is the instantaneous beat frequency. $\Delta\omega_0$ is the frequency difference between the injected signal and the free-running oscillator, which satisfies

$\frac{\Delta\omega_0}{\omega_0} \ll \frac{1}{2Q}$. When the oscillator locks to the external injection signal, the beat frequency vanishes, resulting in the *locking condition*

$$\sin(\alpha) = 2Q\frac{V_0}{V_{inj}}\frac{\Delta\omega_0}{\omega_0}. \tag{2}$$

Since the values of $\sin(\alpha)$ can only be between -1 and $+1$, the maximum locking range of the oscillator is given by

$$\frac{V_{inj}}{V_0} > 2Q|\frac{\Delta\omega_0}{\omega_0}|. \tag{3}$$

B. Semi-analytical Solution Based On Nonlinear Phase Macromodel

Based on the nonlinear phase macromodel proposed in [2], a semi-analytical method is presented for predicting injection locking in oscillators [7]. The oscillator's maximum locking range is governed by equation

$$|\frac{\Delta\omega_0}{\omega_0}| < \eta A_{inj}, \tag{4}$$

where $\Delta\omega_0$ is the frequency difference between injection signal and the oscillator's free-running frequency ω_0, A_{inj} is the injection amplitude and η is the locking factor that can be calculated by solving

$$\eta = \max_{t_0=0\to 1}\int_0^1 v_1(\frac{t+t_0}{f_0})\sin(2\pi t)dt. \tag{5}$$

where $v_1(t)$ is the PPV of oscillators that can be calculated by various numerical methods [2], [3]. When the PPV is calculated, this method can be applied to analyze injection locking in any oscillators, without being limited to harmonic oscillators. However, the calculation of PPVs requires full circuit description at SPICE-level. Thus, this method is not as convenient as traditional analytical solutions.

III. Analytical Solutions For LC and Ring Oscillators

The numerical injection locking method in Section II-B is not very convenient, since it requires numerical calculation of the PPV. If we can derive the analytical PPV of the oscillator under some reasonable simplification and substitute it into (5), we can develop analytical solutions for predicting injection locking in any oscillators. In this section, we derive analytical injection locking equations for the LC and 3-stage ring oscillators. Similar methods can be applied to derive analytical solutions for ring oscillators with more than 3 stages.

A. LC Oscillator

The block diagram of a typical LC oscillator is shown as Figure 1. The general ODE equation for this LC oscillator can be expressed as

$$\boxed{\begin{aligned} \frac{d}{dt}v(t) &= -\frac{v(t)}{RC} - \frac{i(t)}{C} - \frac{f(v(t),i(t))}{C} \\ \frac{d}{dt}i(t) &= \frac{v(t)}{L}, \end{aligned}} \tag{6}$$

where C and L is the effective capacitance and inductance of the resonant tank of the LC oscillator, R is the load resistance, while $f(v(t),i(t))$ is the negative resistor that determines the oscillator's oscillation amplitude. Using vector notation, (6) can be expressed as

$$\dot{x}(t) = Gx(t) + F(x(t)), \tag{7}$$

where $x(t) = [v(t), i(t)]^T$, $F(x(t)) = [-\frac{f(v(t),i(t))}{C}, 0]^T$, and

$$G = \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}.$$

1) Analytical PPV for LC Oscillator: If the Q factor of the oscillator is reasonably high, the steady state of the LC oscillator can be approximated as

$$x_s(t) = \begin{bmatrix} v(t) \\ i(t) \end{bmatrix} = \begin{bmatrix} V_C\cos(w_0 t+\theta) \\ I_L\sin(w_0 t+\theta) \end{bmatrix} \tag{8}$$

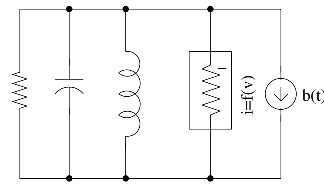


Fig. 1. A simple LC oscillator.

by ignoring some high order harmonics. Here, I_L is the amplitude of the current in the inductor L, V_C is the amplitude of the voltage swinging over the capacitor C, ω_0 is the free-running frequency of the oscillator, and we have relationship:

$$I_L = V_C\omega_0 C, \qquad V_C = I_L\omega_0 L. \tag{9}$$

By introducing new state variable $p = [I_L, \theta]^T$, the steady state of the oscillator can be expressed as

$$x_s(p,t) = \begin{bmatrix} \frac{I_L}{\omega_0 C}\cos(w_0 t+\theta) \\ I_L\sin(w_0 t+\theta) \end{bmatrix} \tag{10}$$

If the oscillator is perturbed by the noise signal $B(t) = [i_n(t), v_n(t)]^T$, I_L and θ will change slowly with time. So the solution of (7) can be given by

$$x(t) = x_s(p(t), t). \tag{11}$$

Substitute (11) in (7), we have

$$\frac{\partial x_s}{\partial p}\dot{p}(t) + \frac{\partial x_s}{\partial t} = Gx(t) + F(x(t)) + \begin{bmatrix} \frac{i_n(t)}{C} \\ \frac{v_n(t)}{L} \end{bmatrix} \tag{12}$$

$$\Rightarrow \frac{\partial x_s}{\partial p}\dot{p}(t) = \begin{bmatrix} \frac{i_n(t)}{C} \\ \frac{v_n(t)}{L} \end{bmatrix} \tag{13}$$

$$\Rightarrow \dot{p}(t) = \begin{bmatrix} \dot{I}_L \\ \dot{\theta} \end{bmatrix} = J^{-1}\begin{bmatrix} \frac{i_n(t)}{C} \\ \frac{v_n(t)}{L} \end{bmatrix}, \tag{14}$$

where

$$\begin{aligned} J^{-1} &= \frac{\partial x_s}{\partial p} \\ &= \begin{bmatrix} \frac{1}{\omega_0 C}\cos(\omega_0 t+\theta) & -\frac{I_L}{\omega_0 C}\sin(\omega_0 t+\theta) \\ \sin(\omega_0 t+\theta) & I_L\cos(\omega_0 t+\theta) \end{bmatrix}^{-1} \\ &= \frac{1}{I_L}\begin{bmatrix} I_L\omega_0 C\cos(\omega_0 t+\theta) & I_L\sin(\omega_0 t+\theta) \\ -\omega_0 C\sin(\omega_0 t+\theta) & \cos(\omega_0 t+\theta) \end{bmatrix}. \end{aligned} \tag{15}$$

Substitute (15) in (14), the phase shift due to the perturbation can be calculated by solving equation:

$$\dot{\theta} = \frac{1}{I_L}[-\omega_0 C\sin(\omega_0 t+\theta), \cos(\omega_0 t+\theta)]\begin{bmatrix} \frac{i_n(t)}{C} \\ \frac{v_n(t)}{L} \end{bmatrix} \tag{16}$$

$$\Rightarrow \dot{\alpha}(t) = [-\frac{1}{I_L}\sin(\omega_0 t+\theta), \frac{1}{V_C}\cos(\omega_0 t+\theta)]B(t), \tag{17}$$

where $\alpha(t)$ is the phase deviation of the oscillator, which has units of time. (17) has the exact same form as the phase equation in [2], so the PPV of the LC oscillator is

$$\mathrm{PPV}(t) = [-\frac{1}{I_L}\sin(\omega_0 t+\theta), \frac{1}{V_C}\cos(\omega_0 t+\theta)]^T. \tag{18}$$

2) Analytical Injection Locking Equation For LC Oscillator: Assuming a sinusoid current signal $i_n(t) = I_i\sin(\omega_1 t)$ is injected into the LC oscillator, the corresponding PPV for this injected signal is

$$v_1(t) = -\frac{1}{I_L}\sin(\omega_0 t+\theta).$$

Substituting $v_1(t)$ and $i_n(t)$ into (5), we have

$$\begin{aligned}\eta &= \max_{t_0=0\to 1}\int_0^1 -\frac{1}{I_L}\sin(2\pi(t+t_0))\sin(2\pi t)dt \\ &= \frac{1}{2I_L}.\end{aligned} \tag{19}$$

So the maximum locking of the LC oscillator can be derived by substituting (19) into (4):

$$|\frac{\Delta\omega_0}{\omega_0}| < \frac{1}{2}\frac{I_i}{I_L}, \tag{20}$$

where I_i is the amplitude of the injected current and I_L is the amplitude of the current in the inductor L.

Using similar method, we derive the equation for voltage injection as

$$|\frac{\Delta\omega_0}{\omega_0}| < \frac{1}{2}\frac{V_i}{V_C}, \tag{21}$$

where V_i is the amplitude of the injected voltage and V_C is the amplitude of the voltage swinging over the capacitor C.

3) Comparing Our Solution With Adler's Equation: For the LC oscillator as shown in Figure 1, the Adler equation has the form of

$$|\frac{\Delta\omega_0}{\omega_0}| < \frac{1}{2Q}\frac{I_i}{I_R}, \tag{22}$$

where Q is the quality factor of the oscillator, I_i is the injected current, and I_R is the load current of the oscillator. The reason why there is inconsistency between these two equations is that Adler's equation uses the load current I_R as reference, while our solution use inductor current I_L as reference. Is is obvious from Figure 1 that I_L and I_R has the relation $I_L = QI_R$, so this two solutions are equivalent. Since the inductor current I_L can be easily measured in a circuit, our method can be very convenient since it doesn't require the Q factor.

B. Ring Oscillator

Figure 2 depicts an simple 3-stage ring oscillator. All resistors, capacitors and inverters are assumed to be identical and all inverters are considered to be ideal, with output voltage $\pm A$ and switching threshold zero. In [11], an analytical PPV was derived for this idealized 3-stage ring oscillator.

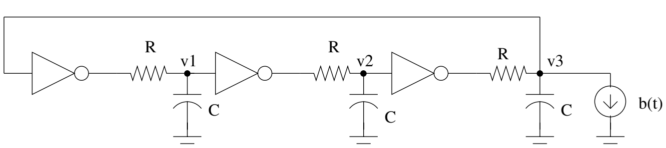


Fig. 2. A simple ring oscillator.

1) Analytical PPV for Ring Oscillator: The ODE equation of the 3-stage oscillator can be expressed as

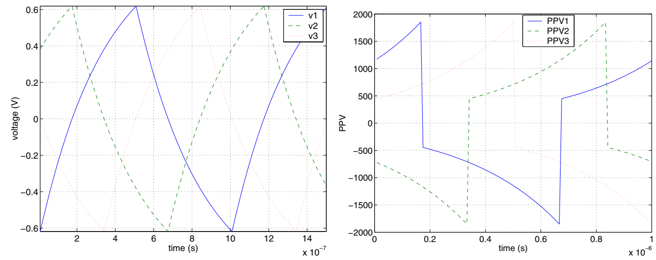
$$\boxed{\begin{aligned}\dot{v}_1(t) &= \frac{f(v_3(t)) - v_1(t)}{RC} \\ \dot{v}_2(t) &= \frac{f(v_1(t)) - v_2(t)}{RC} \\ \dot{v}_3(t) &= \frac{f(v_2(t)) - v_3(t)}{RC},\end{aligned}} \tag{23}$$

where $f(v)$ is the ideal nonlinear inverter, which has the characteristic of

$$f(v) = \begin{cases} -A, & if\ v > 0 \\ A, & otherwise. \end{cases} \tag{24}$$

The steady state response of this idealized ring oscillator is shown [11] to be

$$\begin{aligned} v_1(t) = x(t) &= \begin{cases} A(1-\varphi e^{-\frac{t}{\tau}}), & 0 \le t \le \frac{T}{2} \\ A(-1+\varphi e^{-\frac{t-\frac{T}{2}}{\tau}}), & \frac{T}{2} \le t \le T \end{cases} \\ v_2(t) = x(t-\frac{2T}{3}), &\quad v_3(t) = x(t-\frac{T}{3}), \end{aligned} \tag{25}$$



(a) Steady state waveforms of the ring oscillator.

(b) PPV waveforms of the ring oscillator.

Fig. 3. Steady state waveforms and PPV waveforms of the ring oscillator.

where $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$, τ is the RC time constant and $T = 2.887\tau$ is the period of the ring oscillator. Figure 3(a) depicts the steady state waveforms of the ring oscillator.

By linearizing the ring oscillator equation (23) over its steady state and using the time domain PPV calculation method [2], the PPV of the ring oscillator can be derived [11] to be

$$\text{PPV}(t) = \begin{bmatrix} \text{PPV}_3(t-\frac{2T}{3}) \\ \text{PPV}_3(t-\frac{T}{3}) \\ \text{PPV}_3(t) \end{bmatrix}, \tag{26}$$

where

$$\begin{aligned}\text{PPV}_3(t) &= \frac{R}{A}\frac{1+\varphi^3}{4-2\varphi^3}\left(\varphi + 2\left[-u(t) + (-1+2\varphi^{-1})\,u(t-\frac{T}{2})\right]\right)e^{\frac{t}{\tau}} \\ &\simeq \frac{R}{A}\left(0.4472 - 0.5528u(t-\frac{T}{2})\right)e^{\frac{t}{\tau}},\end{aligned} \tag{27}$$

where $u(t)$ is a unit step function. Figure 3(b) depicts the PPV waveforms of the ring oscillator with $R = 1000\Omega$ and $A = 1V$.

2) Analytical Injection Locking Equation For Ring Oscillator: For a sinusoid current $b(t) = I_i\sin(\omega_1 t)$injected into the ring oscillator, as shown in Figure 2, the maximum locking range can be approximated by substituting (27) into (5):

$$\begin{aligned}\eta &= \max_{t_0=0\to 1}\int_0^1 \text{PPV}_3(\frac{t+t_0}{f_0})\sin(2\pi t)dt \\ &= \max_{t_0=0\to 1}\int_0^1 \frac{R}{A}(0.4472 - 0.5528u(t-\frac{1}{2}))e^{2.887t}\sin(2\pi(t-t_0))dt \\ &= \frac{0.675R}{A}.\end{aligned} \tag{28}$$

$$|\frac{\Delta\omega_0}{\omega_0}| < \eta I_i = \frac{0.675RI_i}{A}. \tag{29}$$

(29) reveals that the maximum locking range of the ring oscillator is proportional to the load resistance R and inversely proportional to the inverters output voltage A. Even though our analytical solution is derived based on the assumption that the switching of the inverter is ideal, our numerical simulation shows that our equation can predict injection locking in ring oscillator very well when the inverter is non-ideal.

IV. Numerical Results

In this section, the analytical equations derived in Section III are applied to predict injection locking of two types of oscillators, LC and 3-stage ring oscillator. For both oscillators, we run full SPICE-level simulations, and compare the results with our analytical solutions. For the ring oscillator case, we simulate the ring oscillator with different switching functions, to investigate the impact of the switching characteristic to the prediction accuracy of our analytical equation. Simulation results show that our analytical equations are able to predict injection locking in oscillators with good accuracy, even for the ring oscillator with non-ideal switching characteristic.

A. LC Oscillator

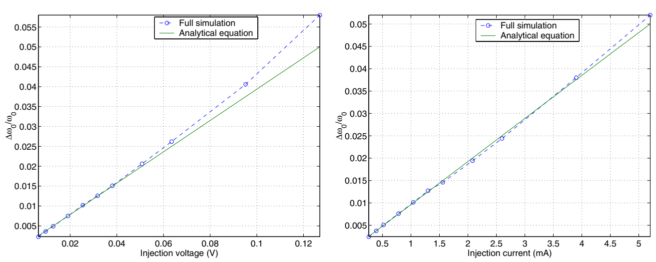
The differential equation for this LC oscillator is

$$\boxed{\begin{aligned} \frac{d}{dt}i(t) &= \frac{v(t)}{L} \\ \frac{d}{dt}v(t) &= -\frac{i(t)}{C} - \frac{v(t)}{RC} - \frac{S}{C}\tanh\left(\frac{G_n}{S}v(t)\right), \end{aligned}} \tag{30}$$

where L, R, and C are the inductance, resistance and capacitance of the LCR tank. S and G_n are parameters of nonlinear negative resistor that enables oscillations. The circuit exhibits autonomous oscillations when $-G_n > 1/R$.

The circuit was simulated with following parameters: $L = 2.5 \times 10^{-8}/(2\pi)\ H,\ C = 4 \times 10^{-11}/(2\pi)\ F,\ R = 100\ \Omega,\ S = 1/R$ and $G_n = -10/R$. With these selected parameters, the oscillator has the Q factor of 4, the LC tank has a resonant frequency of 1 GHz, and the oscillator has an operating voltage of $1.2V$ and an operating current of $52mA$.

We run full SPICE-level simulation to detect the maximum locking range of the LC oscillator under different injection strengths and compare the results with our analytical equations (20) and (21). The results are shown in Figure 4. Our analytical equations produce perfect matches with full SPICE-level simulation for this LC oscillator under both the voltage and current injection.



(a) Voltage injection. (b) Current injection.

Fig. 4. Maximum locking range of the LC oscillator under different perturbations.

B. Ring Oscillator

The three-stage ring oscillator is described by the following differential equations:

$$\boxed{\begin{aligned} \dot{v}_1(t) &= -\frac{v_1}{RC} + \frac{\tanh(-G_m v_3(t))}{RC} \\ \dot{v}_2(t) &= -\frac{v_2}{RC} + \frac{\tanh(-G_m v_1(t))}{RC} \\ \dot{v}_3(t) &= -\frac{v_3}{RC} + \frac{\tanh(-G_m v_2(t))}{RC} \end{aligned}} \tag{31}$$

We choose $C = 2nF$ and $R = 1k\Omega$ in our simulation. G_m determine the switching characteristic of the inverter. The larger the G_m, the smaller the switching threshold of the inverter. When $G_m \Rightarrow \infty$, we get the ideal switching inverter. In our ring oscillator simulation, we simulate the ring oscillator with different G_ms to investigate the prediction accuracy of our analytical equation under different switching thresholds.

We measure the maximum locking range of the ring oscillator with different switching characteristics using full simulation, and plot the results together with our analytical equation in Figure 5. It is obvious that our analytical equation is able to accurately predict injection locking in ring oscillators, even though the switching characteristics is non-ideal.

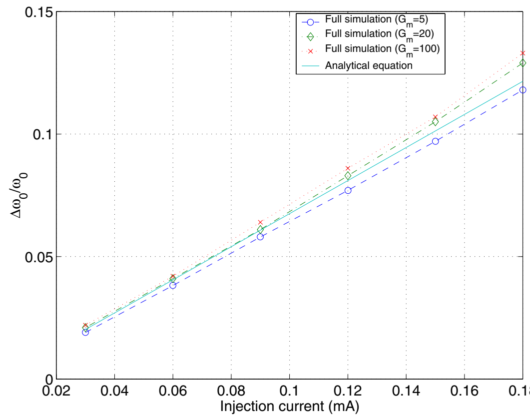


Fig. 5. Maximum locking range of the ring oscillator.

V. Conclusions

We have presented analytical equations for the fast prediction of injection locking in the LC and ring oscillators. We derived the analytical PPVs for the idealized LC and ring oscillators, and developed the analytical injection locking solutions with these PPVs. Our numerical results show that our analytical solutions are able to predict injection locking accurately, even for oscillators with non-ideal characteristics.

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