Efficient AC Analysis of Oscillators using Least-Squares Methods

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***Abstract*— We present a generalization of standard AC analysis to oscillators by exploiting least-squares solution techniques. This provides an attractive alternative to the current practice of employing transient simulation for small signal analysis of oscillators. Unlike phase condition based oscillator analysis techniques, which suffer from numerical artifacts, the least-squares approach of this paper results in a robust and efficient oscillator AC technique. We validate our method on LC and ring oscillators, obtaining speedups of 1-3 orders of magnitude over transient simulation, and 4-6× over phase-condition-based techniques.**

I. Introduction

Oscillators — such as voltage-controlled oscillators (VCOs), digital clocks, *etc.* — are important building blocks in most electronic systems. Analysis of how oscillators respond to small perturbations is crucial in oscillator design. The effects of perturbations on oscillators, in the form of timing jitter (uncertainty of switching edges) or phase noise, can seriously degrade the electronic system performance, hence are major concerns in oscillator design.

The simulation of oscillators under small perturbations presents unique challenges due to their fundamental property of neutral phase stability. The key difficulty with oscillators is that small perturbations lead to arbitrarily large output changes, making standard small signal analysis (*i.e.*, SPICE-like AC analysis) invalid. As a result, the only alternative has been SPICE-like transient simulation [1, 2]. Transient simulation, however, is not well-suited for small-signal analysis from the standpoint of efficiency and accuracy. This is especially the case for oscillators [3], where very small timesteps are required to achieve reasonably accurate results because of accumulation of phase errors.

Over the past few decades, considerable effort has been devoted towards analytical and numerical understanding of the effects of small perturbations on oscillators (*e.g.*, [3–6]). These approaches have mostly focused on the predicting *phase perturbations* of oscillators, typically by obtaining simplified equations for the phase component alone. Applying Floquet theory (*i.e.*, time-varying small-signal perturbation analysis of periodic systems), Kärtner [4,5] derived a scalar, linear time-varying phase equation for oscillator perturbation. This was generalized to a nonlinear differential equation for phase by Demir et al [3, 6]. While these methods are useful for investigating phase behaviour, they do not provide an efficient means of considering the totality of the oscillator's responses, *i.e.*, including both phase and amplitude components. Approaches that do include amplitude components (*e.g.*, [7]) rely on identifying only a few important amplitude modes. However, full Floquet decomposition is computationally expensive as the system size increases.

In this paper, we present a generalization to oscillators of SPICE-like AC analysis, that results in large speedups over transient simulation. Only a single linear matrix solution is involved per frequency point, just as with traditional AC analysis. An interesting feature of our method is that this matrix solution is followed by a "postprocessing" step, in the form of solving a nonlinear scalar differential equation, to capture fully the effects of frequency and phase modulations.

Just as normal AC analysis requires a prior DC solution [1, 2], our method requires a steady-state solution of the oscillator as a base for its nonlinear, time-varying perturbation analysis. Typical oscillator steady state methods, such as harmonic balance [8–10] and shooting (*e.g.*, [8, 11]), rely on adding a "phase condition" equation to remove the ambiguity in phase caused by an underdetermined equation system. Similar disambiguation is also needed in our oscillator AC analysis; however, the use of phase condition equations can cause various numerical artifacts, as we describe later.

A key differentiator of this work is the use of least-squares (LS) solution techniques to solve underdetermined systems without requiring any phase conditions. This resolves the phase ambiguity issue by choosing minimum norm solutions, first solving for a particular solution and then subtracting out null space components. The solutions obtained in this way feature superior smoothness characteristics, resulting in considerable additional robustness and accuracy. In addition, smoothness makes it possible to take large timesteps during the solution of the scalar nonlinear differential equation in postprocessing, which is the most expensive computation in the entire AC analysis, resulting in additional speedups over the phase-condition based approaches.

We demonstrate our LS-based oscillator small signal analysis in detail on LC and ring oscillators. As noted above, results obtained from our method show superior smoothness characteristic compared to those from carefully chosen, "good" phase conditions (which are not easy to find and not uniformly applicable to all cases). All results are in good agreement with full transient simulation results, but the LS-based oscillator AC method provides speedups of 1–3 orders of magnitude. Furthermore, modest speedups of 4-6× are also obtained over phase-condition based oscillator AC.

The remainder of the paper is organized as follows. In Section II, we discuss oscillator AC analysis with the use of phase conditions, based on the generalized multitime partial differential equation (GeMPDE) formulation. In Section III, we demonstrate the smoothness problems that arise with phase condition based oscillator AC analysis. In Section IV, we present the least squares based oscillator AC approach. In Section V, we present validations of the new technique on LC and ring oscillators.

II. Background: GeMPDE based Oscillator AC analysis

Oscillator circuits under perturbation can be described by the DAE system

$$\frac{dq(x)}{dt} + f(x) = Au(t), \tag{1}$$

where $u(t)$ is a small perturbation signal, $x(t)$ is a vector of circuit unknowns (node voltages and branch currents), and A is an incidence matrix that captures the connection of the perturbation to the circuit. It has been shown [3] that small perturbations applied to orbitally-stable oscillators can lead to dramatic changes in output, thus invalidating fundamental assumptions of small-signal analysis. Numerically, this lack of validity leads to rank deficiency in the frequency-domain conversion matrix of oscillators, resulting in a complete breakdown of "normal" AC analysis. It can be proved that the WaMPDE formulation [12], originally proposed to address efficiency problems when encountering strong frequency modulation (FM) in oscillators, succeeds in correcting the problem at DC but fails to do so at all other harmonics. A more generalized form of MPDE (GeMPDE) can solve the problem completely, at all frequencies. In the interests of space, the reader is requested to refer to [12, the Appendix], for details.

The special case of the GeMPDE that is useful for oscillator AC analysis is:

$$\left[\frac{\partial}{\partial t_1} + \hat{\omega}(t_1, t_2)\frac{\partial}{\partial t_2}\right] q(\hat{x}) + f(\hat{x}) = b(t) = Au(t_1). \tag{2}$$

Linearization of the above GeMPDE formulation around the steady state solution $(x^*(t_2), \omega_0)$ (ω_0 is oscillator's free-running frequency), followed by Laplace transform on t_1 and Fourier expansion on t_2, produces a frequency-domain discretized system:

$$\underbrace{\left[\left(\overset{\text{FD}}{\Omega}(s)\mathbb{T}_{C(t_2)}+\mathbb{T}_{G(t_2)}\right), \quad \mathbb{T}_{\dot{q}^*(t_2)}\right]}_{\overset{\text{HB}}{\mathbb{J}}_{Ge(s)}} \begin{pmatrix}\overset{\text{FD}}{\mathbb{V}}_{\Delta X}(s)\\ \overset{\text{FD}}{\mathbb{V}}_{\Delta\omega}(s)\end{pmatrix} = \overset{\text{FD}}{\mathbb{V}}_A U(s). \tag{3}$$

where

$$\mathbb{T}_{C(t_2)} = \begin{pmatrix} & \vdots & \vdots & \vdots & \\ \cdots & C_0 & C_{-1} & C_{-2} & \cdots \\ \cdots & C_1 & C_0 & C_{-1} & \cdots \\ \cdots & C_2 & C_1 & C_0 & \cdots \\ & \vdots & \vdots & \vdots & \end{pmatrix}$$

$$\mathbb{T}_{G(t_2)} = \begin{pmatrix} & \vdots & \vdots & \vdots & \\ \cdots & G_0 & G_{-1} & G_{-2} & \cdots \\ \cdots & G_1 & G_0 & G_{-1} & \cdots \\ \cdots & G_2 & G_1 & G_0 & \cdots \\ & \vdots & \vdots & \vdots & \end{pmatrix}$$

$$\overset{\text{FD}}{\Omega} = j\omega_0 \begin{pmatrix} \ddots & & & & \\ & -I & & & \\ & & 0I & & \\ & & & I & \\ & & & & \ddots \end{pmatrix}$$

$$\overset{\text{FD}}{\Omega}(s) = \overset{\text{FD}}{\Omega} + sI$$
$$\overset{\text{FD}}{\mathbb{V}}_{\Delta X}(s) = [\cdots, \Delta X_{-1}^T, \Delta X_0^T, \Delta X_1^T, \cdots]^T$$
$$\overset{\text{FD}}{\mathbb{V}}_{\Delta\omega}(s) = [\cdots, \Delta \omega_{-1}^T, \Delta \omega_0^T, \Delta \omega_1^T, \cdots]^T$$
$$\overset{\text{FD}}{\mathbb{V}}_{A} = [\cdots, 0, A^T, 0, \cdots]^T.$$

$\overset{\text{HB}}{\mathbb{J}}_{Ge}(s)$ is a rectangular matrix of size $nN \times (n+1)N$, where n is the number of circuit unknowns and N is the number of terms in truncated Fourier series. It can be proved that $\overset{\text{HB}}{\mathbb{J}}_{Ge}(s)$ is full rank at any frequency.

To solve (3), N more equations, termed phase conditions, can be added to the system. The phase condition rows that augment $\overset{\text{HB}}{\mathbb{J}}_{Ge}(s)$ need to satisfy two conditions:

1) must be full rank themselves,
2) in addition to making the entire augmented Jacobian matrix full rank.

The transfer function is then calculated after augmentation with these phase conditions.

Once the transfer function is available, the quantities $\overset{\text{FD}}{\mathbb{V}}_{\Delta X}(s)$ and $\overset{\text{FD}}{\mathbb{V}}_{\Delta\omega}(s)$ at different frequencies can be obtained. Multitime waveforms of Δx and $\Delta\omega$ at a given frequency can then be further obtained via the inverse discrete Fourier transform. Finally, time-domain phase variations can be recovered by the phase-frequency relation [12]:

$$\frac{d\Delta\phi(t)}{dt} = \hat{\Delta\omega}(t, \omega_0 t + \Delta\phi(t)). \tag{4}$$

The one-time form of amplitude variation $\Delta x(t)$ can also be recovered using:

$$\Delta x(t) = \hat{\Delta x}(t, \phi(t)), \tag{5}$$

where $\phi(t) = \omega_0 t + \Delta\phi(t)$. The overall solution of the oscillator is given by

$$x(t) = x^*(\phi(t)) + \hat{\Delta x}(t, \phi(t)), \tag{6}$$

where x^* is the steady state oscillatory solution.

III. Problem with phase condition based Oscillator AC analysis

Theoretically, there is considerable apparent freedom in choosing phase conditions, as long as they satisfy two conditions mentioned in Section II. Unfortunately, many such phase conditions are not efficient or not capable of generating useful information from the standpoint of small-signal analysis.

The key to understanding this problem is that the unique solution $(\Delta x, \Delta\omega)$ obtained by adding constraints (*i.e.*, the phase conditions) are essentially an arbitrary choice, leading to unphysical artifacts such as significant non-smoothness. While it is possible to find "good" phase conditions that avoid these problems, good conditions is very problem specific and their discovery is difficult, hence of limited general value for enabling a robust, general-purpose algorithm. The smoothness of the bivariate frequency solution is especially important for calculating phase variations (4), the very first step in recovering overall solutions, and the most computationally expensive step. More specifically, if the bivariate form of frequency variation ($\Delta\omega$) obtained from transfer function is not smooth, *i.e.*, there are lots of undulations in the multi-time waveform, very small time steps must be taken in order to generate useful phase variations using (4). In some cases, the change on one time scale, or both time scales, is so rapid that no useful information can be obtained even when very small time steps are used. Recall that (4) is the only ODE that needs to be solved in oscillator AC analysis, *i.e.*, it is the main computation (other computations only involve linear matrix solution and interpolation). The step size taken in solving (4) essentially determines the speed of the entire AC analysis.

Furthermore, other calculations, such as amplitude variations and overall solutions ((5), (6)) depend on the results of phase variations. Oscillator AC analysis becomes impractical if the phase variation calculated is not useful.

To demonstrate this in detail, we use the phase condition equations:

$$\frac{\partial}{\partial t_1}\hat{x}_l + \omega(t_1, t_2)\frac{\partial}{\partial t_2}\hat{x}_l = \frac{\partial x_{ls}(t_2)}{\partial t_2}, \tag{7}$$

where l is a fixed integer. $\hat{x}_l$ denotes the l^{th} element of $\hat{x}$, while x_{ls} is the l^{th} element of the steady state solution $x_s(t_2)$.

By linearizing around (x_l^*, ω_0) and expanding the t_2 dependence in Fourier series, we obtain:

$$\underbrace{\left[\overset{\text{FD}}{\Omega}(s)\mathbb{T}_{e_l^T}, \quad \mathbb{T}_{\dot{x}_l^*(t_2)}\right]}_{P} \begin{pmatrix}\overset{\text{FD}}{\mathbb{V}}_{\Delta X}(s)\\ \overset{\text{FD}}{\mathbb{V}}_{\Delta\omega}(s)\end{pmatrix} = 0. \tag{8}$$

The transfer function is calculated using

$$\begin{aligned}\overset{\text{FD}}{\mathbb{V}}_{H}(s) &= \begin{pmatrix}\overset{\text{FD}}{\mathbb{V}}_{\Delta X}(s)\\ \overset{\text{FD}}{\mathbb{V}}_{\Delta\omega}(s)\end{pmatrix} / U(s) \\ &= \begin{bmatrix} \left(\overset{\text{FD}}{\Omega}(s)\mathbb{T}_{C(t_2)}+\mathbb{T}_{G(t_2)}\right), & \mathbb{T}_{\dot{q}^*(t_2)} \\ \overset{\text{FD}}{\Omega}(s)\mathbb{T}_{e_l^T}, & \mathbb{T}_{\dot{x}_l^*(t_2)} \end{bmatrix}^{-1} \begin{pmatrix}\overset{\text{FD}}{\mathbb{V}}_{A}\\ z\end{pmatrix}.\end{aligned} \tag{9}$$

where $z = [0, ..., 0]^T$ (size N).

These phase conditions above satisfy the conditions mentioned in Section II, as demonstrated in Figure 1. For illustration, we use a simple LC oscillator with a negative resistor. The circuit is perturbed by a current source in parallel with the inductor.

Figure 2(a) shows the bivariate form of frequency variations under a perturbation of $4 \times 10^{-5} sin(1.03 w_0 t)$, using the above phase conditions. As can be seen, it has many undulations in the t_2 time scale. In this case, the rate of variation is so rapid that no useful phase condition can be solved for, as shown in Figure 2(b). If we continue to solve for amplitude and overall solutions, we obtain the results shown in Figure 3 and 4. Note that bivariate amplitude variation also shows rapid

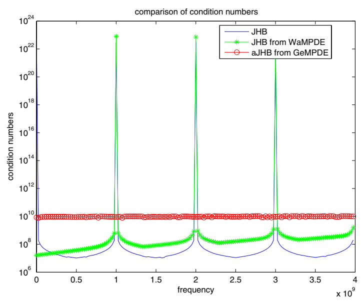
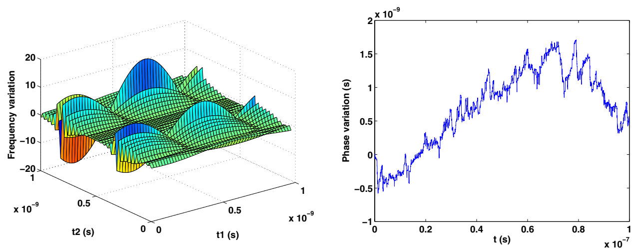
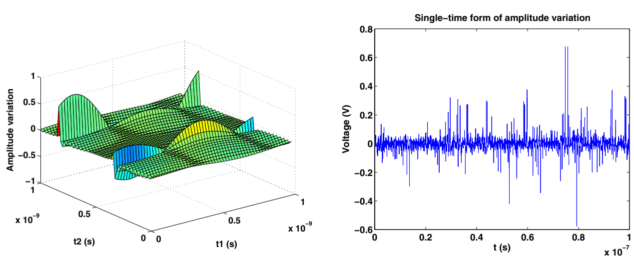


Fig. 1. Condition numbers: original Jacobian matrix (solid line), augmented Jacobian from WaMPDE (*), and augmented Jacobian from GeMPDE (o). The frequency of LC oscillator is 1GHz. We use 61 harmonics.



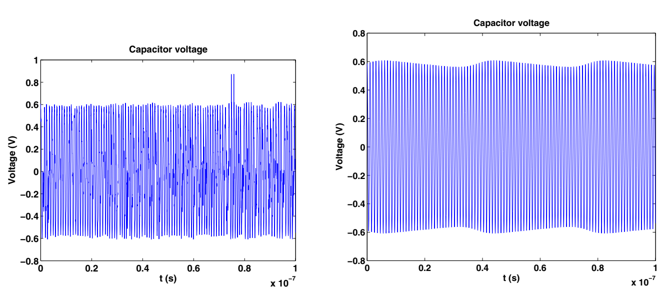
(a) Multi-time frequency variation (b) Single-time phase variation

Fig. 2. Frequency and phase variations when the perturbation current is $4 \times 10^{-5} sin(1.03w_0 t)$. The figure shows the results for 100 cycles.



(a) Multi-time amplitude variation (b) Single-time amplitude variation

Fig. 3. Amplitude variations.



(a) Result from AC analysis (b) Transient simulation result

Fig. 4. Comparison of the result from AC analysis and transient simulation.

changes along the t_2 time scale. For comparison, the full simulation result is also shown in Figure 4. It is clear that this particular phase condition based GeMPDE AC analysis generates invalid results.

However, the slightly different phase conditions

$$\omega(t_1, t_2)\frac{\partial}{\partial t_2}\hat{x}_l = \frac{\partial x_{ls}(t_2)}{\partial t_2} \tag{10}$$

provide valid solutions that are in good agreement with full simulation results, as shown in Section V-A. The results from these ”good” phase conditions, however, are not as good as those from the MLS method proposed in the next section of this paper, in that the corresponding bivariate frequency is not as smooth as that from MLS, as will be shown in Section V. In summary, the practical utility of oscillator AC analysis depends heavily on the phase conditions added. In some cases, “good” phase conditions for certain examples may generate invalid results for other examples. It is difficult if not impossible to find generically “good” phase conditions that work well for all examples.

IV. Solving for Transfer Function by Least Squares

There is more than one solution of (3) since $\overset{\text{HB}}{\mathbb{J}}_{Ge}(s)$ is rectangular but full rank, with N with more columns than rows. To obtain a unique solution, we choose the solution with minimum norm that satisfies (3), defined as the Minimum Least Square (MLS) solution. This MLS solution can be obtained[1] by first finding a particular solution of (3) and then subtracting the projection of this particular solution on the null space (projection onto the null space is the same as that on the solution space, since the solution space is just a constant shift of the null space). Figure 5 shows that such a solution is a vector that is orthogonal to the solution space, *i.e.*, it is of minimum norm.

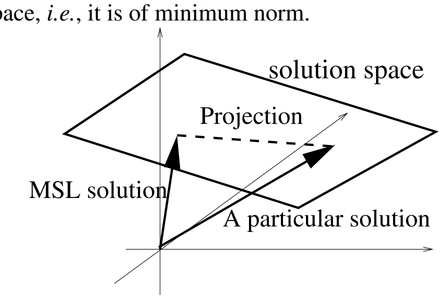


Fig. 5. Illustration of minimum least squares solution.

More specifically, the MLS solution can be found as follows:

1) Find a particular solution of (3), denoted as x_{par}. This can be done by setting all free variables to 0s and solving (3). (Free variables are defined as the variables without pivots in $\overset{\text{HB}}{\mathbb{J}}_{Ge}(s)$ when Gaussian elimination is performed.)
2) Obtain the null space of (3), *i.e.*, find linearly independent vectors that span the null space. These vectors can be obtained by setting one of free variables to 1 and the rest of them to 0s, and then solving (3). Then the general solution of (3) is a constant shift, the particular solution, of the null space.
3) Obtain an orthonormal basis of the null space. The standard Gram-Schmidt is used to convert the basis found above to an orthonormal basis. The orthonormal basis is denoted as $\{n^1, n^2, ..., n^N\}$.
4) Subtract the projection of the particular solution on the null space. This can be done by subtracting all components of the

[1] Any MLS solver, such as ones based on computing the singular value decomposition of $\overset{\text{HB}}{\mathbb{J}}_{Ge}(s)$ or projecting the solution space into a smaller dimension, may be used for this step; however, these do not, in general, exploit the structure of the matrix, hence have cubic solution complexity in the size of the matrix. The procedure outlined here, in constrast, exploits the small dimension of the null space to reduce the complexity of MLS solution to almost linear.

particular solution on the null space basis from the particular solution, *i.e.*,

$$x_{MLS} = x_{par} - \sum_{i=1}^{N} (x_{par}, n^i) n^i. \qquad (11)$$

Here $(.,.)$ denotes the dot or inner product.

V. Applications and Validation

In this section, we apply the MLS-base GeMPDE small signal analysis to several oscillators. Comparisons with phase condition based AC analysis confirm that our method generates "better" or smoother solutions, resulting in further speedups since large time step can be taken. All simulation were performed using MATLAB on an 2.4GHz, Athlon XP-based PC running Linux.

A. 1GHz negative-resistance LC Oscillator

A simple 1GHz LC oscillator with a negative resistor is shown in figure 6. At the steady state, the amplitude of the inductor current is 1.2mA and the capacitor voltage is about 0.585V.

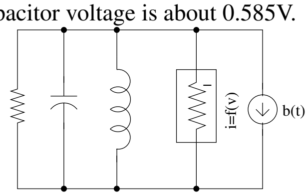
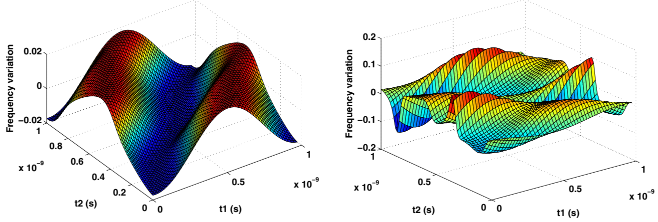


Fig. 6. A simple 1GHz LC oscillator with a negative resistor

The circuit is perturbed by a current source in parallel with the inductor. Figure 7 shows the multi-time form of the local frequency, under a perturbation of $4 \times 10^{-5} sin(1.03 w_0 t)$, solved by MLS and by adding "good" phase conditions, respectively. It can be seen that the MLS solution is much smoother than the solution obtained by adding phase conditions, resulting in a much smoother phase variation recovered from the bivariate form of frequency (using (4)), as shown in Figure 8. As a result, by using MLS, it becomes possible to use much larger time steps to solve the nonlinear scalar equation (4), which accounts for the main computational cost of oscillator AC analysis.



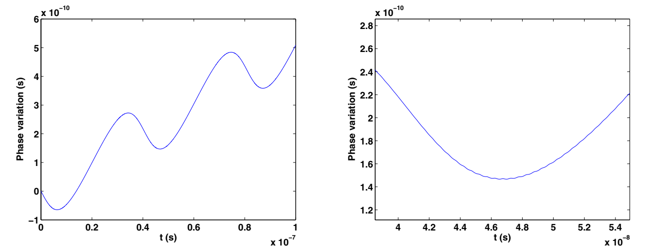
(a) Frequency variations solved by MLS (b) Frequency variations solved by adding phase conditions

Fig. 7. Bivariate form of frequency variation when the perturbation current is $4 \times 10^{-5} sin(1.03 w_0 t)$.

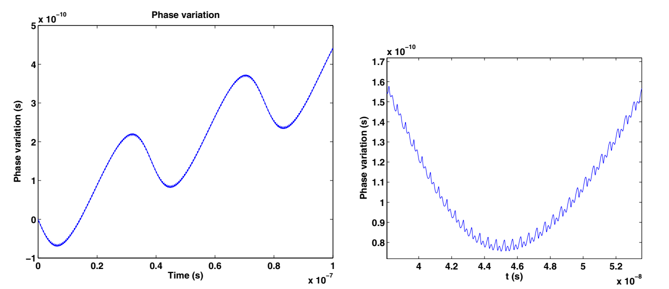
Both multi-time and recovered one-time forms (using MLS methods) of amplitude variation of the capacitor voltage are shown in Figure 9. The capacitor voltage waveform recovered from phase and amplitude variations from MLS is compared with full transient simulation in Figure 10 to confirm the validity of our method. As can be seen, the results from our method match full simulation perfectly. A further speedup of 4-5 over phase condition based method is obtained, resulting in a total speedup of about 90 over full simulation.

B. 3 Stage Ring Oscillator

A 3 stage oscillator with identical stages is shown in Figure 11. The oscillator has a natural frequency of 1.53×10^5Hz. The amplitude of steady state load current is about 1.2mA. The circuit is perturbed by a

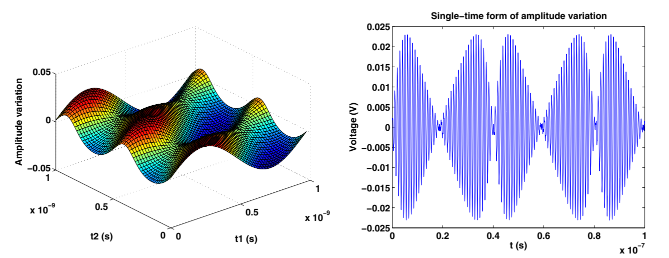


(a) Phase variation solved by MLS (b) Detailed phase variation solved by MLS



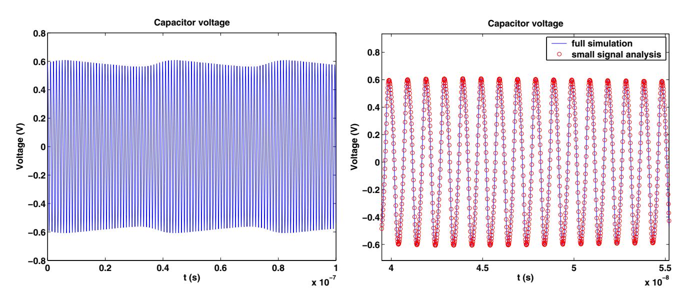
(c) Phase variation solved by adding phase conditions (d) Detailed phase variation solved by adding phase conditions

Fig. 8. Phase variations. The simulation length is 100 cycles (of the oscillator's free-running period).



(a) Bivariate form of amplitude variations (b) Single-time form of amplitude variations

Fig. 9. Amplitude variation solved by MLS.



(a) Result recovered from small signal analysis (b) Detailed comparison (zoom in)

Fig. 10. Comparison of results from small signal analysis and full transient simulation.

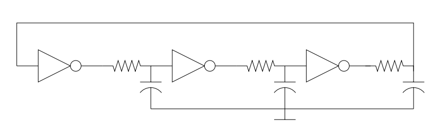
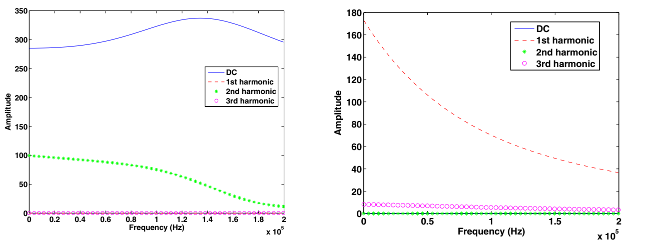


Fig. 11. A 3 stage oscillator with identical stages

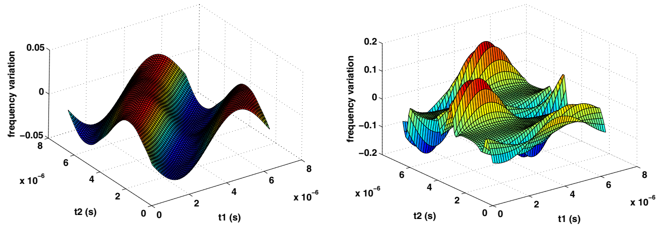
current source, which is connected in parallel with the load capacitor at node 1 and has much smaller current compared to the steady state load current.

Figure 12 shows frequency sweeps akin to standard AC analysis for both the capacitor voltage and the local frequency. Figures 13-14 show the frequency and the resulting phase variations, using MLS and phase condition based approaches, respectively. It is clear from detailed comparison that MLS provides better and smoother solution. The corresponding amplitude variation at node 1 is shown in Figure 15(a). The total waveform at node 1 is compared with full transient simulation in Figure 15(b). Again, we see perfect agreement between results from MLS and SPICE-like simulation, with a speedup of about 100 in this case. Compared to a speedup of 20 obtained by phase condition based method, our MLS based technique gains an additional speedup of about $5\times$ for this example also.



(a) Harmonic transfer functions at node 1 (b) Harmonic transfer functions of the local frequency

Fig. 12. Harmonic transfer functions: the frequency sweeps from DC to $2 \times 10^5 Hz$



(a) Frequency variation solved by MLS (b) Frequency variation by phase condition based method

Fig. 13. Bivariate form of frequency variations when the perturbation current is $5 \times 10^{-5} sin(1.04 w_0 t)$.

C. 4GHz Colpitts LC Oscillator

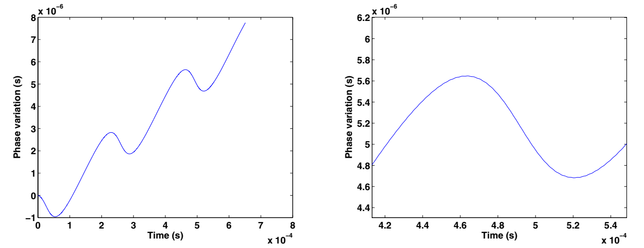
A Colpitts LC oscillator is shown in Figure 16. The free-running frequency of the oscillator is approximately 4GHz.

We perturb the oscillator with a small sinusoidal voltage source ($2 \times 10^{-3} sin(1.02 w_0 t)$) in series with L1. Figure 17 show frequency sweeps for both the current through L1 and the local frequency. Figure 18 shows the local frequency and phase variation from the MLS. Comparisons of phase variations recovered from bivariate frequency variations using different methods are shown in Figure 19. It is clear that results from MLS are much smoother than those from the phase condition based approach.

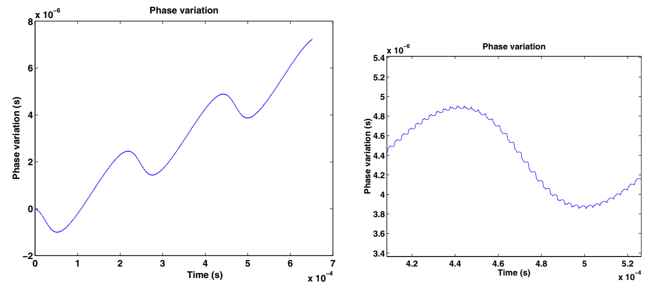
The amplitude variation of the current through L1 obtained from MLS is shown in Figure 20(a). The total waveform of the current through L1 is shown in Figure 20(b) (The comparison with transient simulation is omitted due to the space limit). We obtain a speedup of around 6 over the phase condition based method, resulting in a total speedup of about 600 over transient simulation.

VI. Conclusions

We have presented a least-squares based approach for performing AC analysis of oscillators. Unlike previous approaches to small-perturbation analysis of oscillators, our methods captures all amplitude

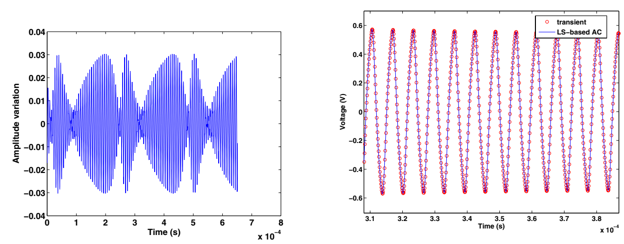


(a) Phase variation by MLS (b) Detailed phase variation by MLS



(c) Phase variation by phase condition based method (d) Detailed phase variation by phase condition based method

Fig. 14. Phase variations. The figure shows simulation result for 100 cycles.



(a) Single-time form of amplitude variations (b) Detailed comparison of the result from LS-base AC analysis and transient simulation

Fig. 15. Results from LS-base AC analysis and transient simulation.

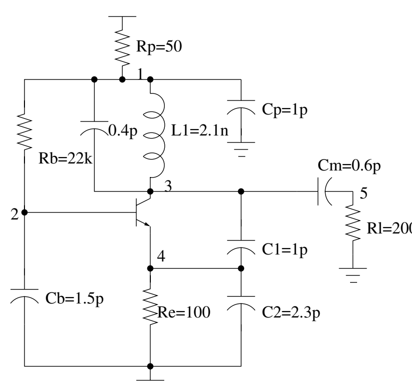
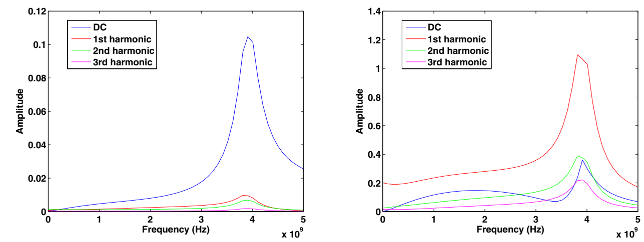
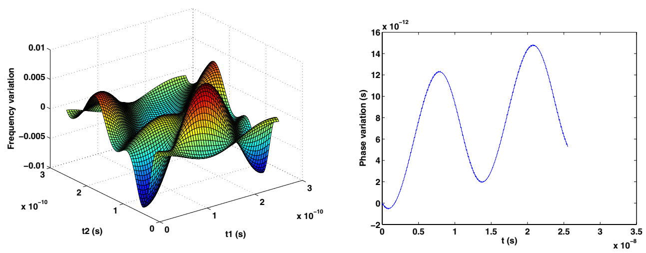


Fig. 16. A 4GHz Colpitts LC oscillator



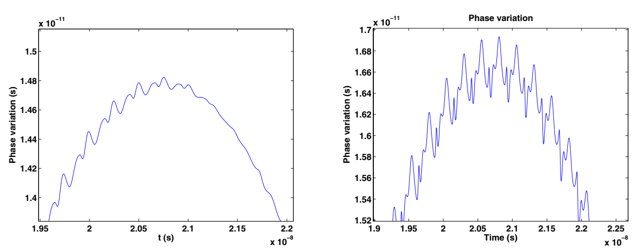
(a) Harmonic transfer functions of the current through L1 (b) Harmonic transfer functions of the local frequency

Fig. 17. Harmonic transfer functions: the frequency sweeps from DC to $5 \times 10^9 Hz$



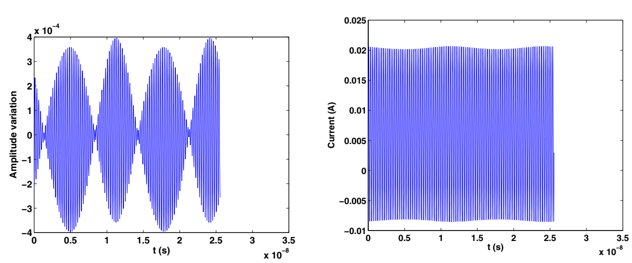
(a) Bivariate form of frequency variations from MLS (b) Phase variations.

Fig. 18. Frequency variation and phase variation from MLS when the perturbation current is $2 \times 10^{-3} sin(1.02 w_0 t)$.



(a) Detailed phase variations from MLS (b) Detailed phase variations from phase condition based method

Fig. 19. Comparison of phase variations.



(a) One-time solution of amplitude variation (b) Result recovered MLS small signal analysis

Fig. 20. Result from MLS small signal analysis. The figure shows simulation results for 100 cycles.

and phase components of the oscillator's response correctly, while being 1–3 orders of magnitude faster than transient simulation, the only realistic alternative. Our technique also constitutes a significant improvement, in terms of accuracy, speed and robustness, over a closely related alternative that uses phase conditions.

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