An Impulse-Response Based Linear Time-Complexity Algorithm for Lossy Interconnect Simulation

Jaijeet S. Roychowdhury A. Richard Newton Donald O. Pederson

Department of Electrical Engineering and Computer Sciences
University of California, Berkeley

Abstract

In this paper, a linear time-complexity algorithm for lossy transmission line simulation within arbitrary nonlinear circuits is presented. The method operates by storing information about the state of the line at dynamically selected internal points and using an analytical formulation based on impulse responses to predict the line's future behaviour accurately. Previous approaches using impulse responses (based on time-domain convolution) possess quadratic time-complexity, which can lead to long computation times for simulations with many time-points. In the proposed method, integration over space with fixed limits replaces time-domain convolution, eliminating the quadratic time-complexity. The method does not require rational or other approximations of transfer-functions to achieve linear time-complexity nor does it increase the size of the simulator's matrix by more than 2 for each transmission line. Experimental results on industrial circuits indicate that for equivalent or superior accuracy, the state-based method can be faster for simulations of one or more clock or data pulses, with speedups of more than 10 and 50 over the convolution and lumped-RLC methods for the longer simulations.

1 Introduction

Simulating lossy transmission lines within nonlinear circuits is important in the computer-aided design of high-performance systems, particularly of multi-chip modules (MCMs) [1]. In these high-speed digital circuits, fast switching speeds combined with long, lossy interconnections give rise to the need for transmission line models. Fast and accurate simulation techniques are needed during the design and verification stages to ensure that transmission line effects do not affect correct operation. The fact that interconnect in MCMs and ICs are lossy makes their simulation a difficult problem, in contrast to lossless lines for which satisfactory techniques exist [2].

The most common existing technique for lossy line simulation is the lumped-RLC approach [3], in which a line is represented by segments each consisting of lumped R, L and C elements. A variant of this approach, the pseudo-lumped approach [4], uses segments of lossless lines and series resistors. In another technique [5], the irrational transfer functions of the lossy line are approximated by rational functions to obtain a reduced-order model that can be represented by lumped elements. The waveform relaxation based method [6] solves the line's equations in the frequency domain and uses the FFT to switch to and from the time domain. In the convolution method [7, 8, 9], the linearity of the lossy line is exploited by using its impulse-responses for time-domain simulation. Though the convolution technique is accurate and does not use approximations to transfer functions, the execution time of the method rises quadratically with the number of simulation time-points.

In this paper, a simulation technique with linear time-complexity[1] is presented. The technique is currently applicable only to "simple lossy lines" (uniform transmission lines with constant (frequency-independent) resistance, capacitance, inductance and conductance (R, L, C and G) per unit length)[2]. The technique is referred to as the "state-based" method because it utilizes information about the internal state of the transmission line at a given time to solve for the next time-point. As in the convolution technique [9], analytically known impulse responses are used. A key difference is that convolution is eliminated, replaced by an integral in space with fixed limits involving the state of the line. Knowledge of the entire waveforms at the ends of the line is not used as in the convolution technique. The location and number of the sample points used to store the internal state of the line are varied during the simulation to track propagating waves, ensuring accuracy and computational efficiency.

Section 2 contains the formulation of the state-based method and a comparison of its features with those of other methods. Results on sample circuits are presented in Section 3, followed by conclusions in Section 4.

2 State-based Method

2.1 Formulation

The development of the state-based method starts from the Telegrapher Equations [8]:

$$\frac{\partial v}{\partial x} = -\left(L\frac{\partial i}{\partial t} + R\,i\right) \tag{1}$$

[1] The exponent to which the total number of simulation time-points is raised to obtain the execution time, within a constant proportionality factor.

[2] More accurate and complex models ("frequency-varying models") have R, L, C and G varying as functions of frequency to model different effects, such as skin-effect [10] and dielectric dispersion [5]. While the simulation of frequency-varying models is an important problem, Deutsch et. al. [10] have shown that simple lossy line models are adequate for most practical applications today.

$$\frac{\partial i}{\partial x} = -C\frac{\partial v}{\partial t} \tag{2}$$

The parallel conductance G is assumed to be zero for simplicity; all of what follows continues to hold with minor modifications for nonzero G. The above equations hold for x varying between 0 and l, the length of the transmission line. $v(x,t)$ and $i(x,t)$ are the voltage and current at the point x in the line at time t, respectively. Without loss of generality (because Equations 1 and 2 are time-invariant), the assumptions are made that the simulation starts from time 0 and that the next time-point is t. Note that in a simulator, t is actually Δt, the incremental time-step. The inputs to the transmission line come from the port variables $v_1(t) = v(0,t)$, $i_1(t) = i(0,t)$, $v_2(t) = v(l,t)$ and $i_2(t) = -i(l,t)$. These four port variables specify the *boundary conditions* of Equations 1 and 2.

In addition to the boundary conditions which represent the inputs to the line from the external circuit, the internal state of the transmission line also determines the future behaviour of the line. This internal state is stored in the energy-storing distributed inductance and capacitance and is specified by the voltages and currents in the line's interior at time 0, $v_0(x) = v(x,0)$ and $i_0(x) = i(x,0)$. $v_0(x)$ and $i_0(x)$ are the *initial conditions* for Equations 1 and 2. The combination of the Telegrapher Equations and the boundary and initial conditions specify the future behaviour of the line uniquely. The state-based method uses the available initial and boundary conditions at time 0 to find the values of the port variables as well as the new internal state of the line at the next time-point t. The procedure is then repeated using the newly calculated line-state ("resetting" time to 0, so to speak). This is continued until the end of the simulation.

Laplace transforms are taken (in t) of Equations 1 and 2 to arrive at ordinary differential equations in x and s, the Laplace variable:

$$\frac{\partial V}{\partial x} = -(sL+R)\,I + L\,i_0(x) \tag{3}$$

$$\frac{\partial I}{\partial x} = -sC\,V + C\,v_0(x) \tag{4}$$

V and I refer to $V(x,s)$ and $I(x,s)$, the Laplace transformed variables.

For brevity, the mathematical details of the formulation from this point till the final result are omitted, and only a description of the steps involved is given[3]. V and I are replaced by scattering parameter variables through a change of basis, and the resulting equations algebraically manipulated to obtain first-order nonhomogeneous ordinary differential equations. These are solved analytically, boundary conditions are applied, and the scattering parameter variables replaced by voltage and current variables. Following this, integrals over space (that arise during the nonhomogeneous ODE solution) are evaluated analytically, using a piecewise-constant approximation for $v_0(x)$ and $i_0(x)$. Finally, the inverse Laplace transform is applied to obtain the following time-domain constitutive relations:

$$\begin{aligned}&\left[Y_0\, v(x,t) * h_Y(t) + i(x,t)\right] \\ &- \left[Y_0\, v(0,t) * h_{\gamma Y}(x,t) + i(0,t) * h_\gamma(x,t)\right] \\ &= \sum_{j=1}^{n_x} \Big\{ i_{0j}\left[h_{S\gamma Y}(x-x_j,t) - h_{S\gamma Y}(x-x_{j-1},t)\right] \\ &\qquad + Y_0\, v_{0j}\left[h_{S\gamma}(x-x_j,t) - h_{S\gamma}(x-x_{j-1},t)\right]\Big\}\end{aligned} \tag{5}$$

$$\begin{aligned}&\left[Y_0\, v(l,t) * h_{\gamma Y}(l-x,t) - i(l,t) * h_\gamma(l-x,t)\right] \\ &- \left[Y_0\, v(x,t) * h_Y(t) - i(x,t)\right] \\ &= \sum_{j=n_x}^{n_l} \Big\{ i_{0j}\left[h_{S\gamma Y}(x_{j-1}-x,t) - h_{S\gamma Y}(x_j-x,t)\right] \\ &\qquad - Y_0\, v_{0j}\left[h_{S\gamma}(x_{j-1}-x,t) - h_{S\gamma}(x_j-x,t)\right]\Big\}\end{aligned} \tag{6}$$

In the above, the spatial interval $[0,l]$ is partitioned into segments between the points $x_0, x_1, \cdots, x_{n_l}$, with $x_0 = 0$ and $x_{n_l} = l$. Samples (at x_i) of the initial voltage and current states of the line are represented by v_{0i} and i_{0i}. The index n_y is defined by the relation $x_{n_y} = y$ for any $y \in [0,l]$. $*$ denotes the convolution operator; Y_0 is defined to be $\sqrt{\frac{C}{L}}$, and h_Y, $h_{\gamma Y}$, h_γ, $h_{S\gamma Y}$ and $h_{S\gamma}$ are defined as follows:

$$h_Y(t) = e^{-\beta t}\left[\delta(t) + \beta\left\{I_1(\beta t) - I_0(\beta t)\right\}\right] \tag{7}$$

$$\begin{aligned}h_\gamma(x,t) = e^{-\beta t}\Big[&\delta(t-\gamma_0 x) \\ &+ u(t-\gamma_0 x)\frac{\beta\gamma_0 x}{\sqrt{t^2-(\gamma_0 x)^2}} I_1\big(\beta\sqrt{t^2-(\gamma_0 x)^2}\big)\Big]\end{aligned} \tag{8}$$

$$\begin{aligned}h_{\gamma Y}(x,t) = e^{-\beta t}\Big[&\delta(t-\gamma_0 x) + u(t-\gamma_0 x)\,\beta \times \\ &\left\{\frac{t}{\sqrt{t^2-(\gamma_0 x)^2}} I_1\big(\beta\sqrt{t^2-(\gamma_0 x)^2}\big) - I_0\big(\beta\sqrt{t^2-(\gamma_0 x)^2}\big)\right\}\Big]\end{aligned} \tag{9}$$

$$h_{S\gamma}(x,t) = u(t-\gamma_0 x)\, e^{-\beta t}\, I_0\big(\beta\sqrt{t^2-(\gamma_0 x)^2}\big) \tag{10}$$

$$\begin{aligned}h_{S\gamma Y}(x,t) = u(t-\gamma_0 x)\, e^{-\beta t}\Big[&e^{-\beta(t-\gamma_0 x)} \\ &+ \beta\gamma_0 x \int_{\gamma_0 x}^{t} e^{-\beta(t-\tau)} \frac{I_1\left(\beta\sqrt{\tau^2-(\gamma_0 x)^2}\right)}{\sqrt{\tau^2-(\gamma_0 x)^2}}\, d\tau\Big]\end{aligned} \tag{11}$$

In the above, $\beta = \frac{R}{2L}$, $\gamma_0 = \sqrt{LC}$, δ and u are the delta and unit functions, and I_0 and I_1 are the modified Bessel functions of zeroth and first orders respectively.

Equations 5 and 6 are time-domain constitutive relationships for the lossy transmission line at any x. *It is important to note that the convolution operation in the LHS of the equations is performed over only one time-step, from 0 to t and does not extend over the past history of the simulation.* The RHS of the equations depend only on the (known) initial line-state; $v(x,t)$ and $i(x,t)$ can be determined from the initial line-state and the values of the port variables at the

[3] for the details, see [11]

current time-point t. In particular, $v(x,t)$ and $i(x,t)$ at any x are independent of $v(y,t)$ and $i(y,t)$ for any $y \neq x$ in the interior of the line; this implies that a system of simultaneous equations does not have to be solved to determine the line-state at t.

To solve for the port variables at time t, the two equations obtained by substituting $x = l$ in Equation 5 and $x = 0$ in Equation 6 are used to load the circuit-simulator matrix and the right-hand-side vector of excitations. The circuit is then solved by the simulator and the port variables determined at t. For each x chosen to sample the new internal state of the line, Equations 5 and 6 are then used to obtain the voltage and current at time t.

2.2 Features

While the concept of using the internal state of the line is reminiscent of the segmenting approaches, there are three important differences. First, analytical solutions available for delta-function excitations (the impulse responses) are utilized in the state-based method, leading to improved accuracy for all values of the line's electrical parameters. Second, the points at which the internal line-state is sampled are varied dynamically; they are chosen densely in regions where waveforms are fast-varying and sparsely in regions where waveforms vary slowly. Since dynamic variation is impossible in the segmentation techniques, all the sample points have to be densely chosen, i.e., many segments need to be used. Third, segmentation techniques increase the size of the circuit matrix; the interdependence between the unknown internal variables at the present time-point requires their simultaneous solution. This is not the case in the state-based method, where the new state at any internal point is calculated individually and explicitly.

It is instructive to compare the convolution and state-based methods from the aspect of time-complexity, since both use impulse responses. Physically speaking, a transmission line has no mechanism for storing the entire past history of its port variables; instead, at any given time, the internal currents and voltages (the state) are maintained by the distributed inductance and capacitance of the line and this state dictates the future behaviour of the line. The convolution method stores only a history of its port variables. In a sense, the convolution method wastes computation attempting to recreate information more directly available from the internal state.

The main computational effort in the state-based method is the calculation of the line-state at every time-point. The computation involved is however approximately independent of time, depending only on the number of internal samples of the line[4]; hence the linear time-complexity, which implies that for simulation lengths generating more than some N_{crit} time-points, the state-based method is faster than the convolution method. The crucial question is, of course, whether N_{crit} is achieved and exceeded in practical simulations. Examples in Section 3 show that this is indeed so in real-life circuits being simulated for one or more clock or data pulses.

[4] the number of internal samples needed is proportional to the number of sharp edges in the line's internal waveforms; in most circuits, there will be no more than two or three such edges at a given time

3 Experimental Results

Waveform and computation speed comparisons with the lumped-RLC and convolution methods for four circuits are presented in this section.

The first three circuits use digital BJT drivers with output rise-times of 500ps–2ns, connected by lossy interconnect to diode receivers or other BJT drivers. **raytheon1** has one fan-out, **raytheon2** has branching interconnect to three fan-outs and **raytheon3** has a 2-wire multiconductor line; all three use identical interconnect parameters[5]. **mosaic** is a single lossy line driven by a voltage source with series resistance and terminated by clamping diodes; interconnect parameters for **mosaic** were taken from [1]. For the lumped-RLC method, 10 segments per inch were used for the first three circuits, and 4 segments per cm for **mosaic**. Table 1 compares execution times and Fig. 1 depicts a sample output waveform. Figure 2 shows the variation of execution time vs. simulation length for the three methods.

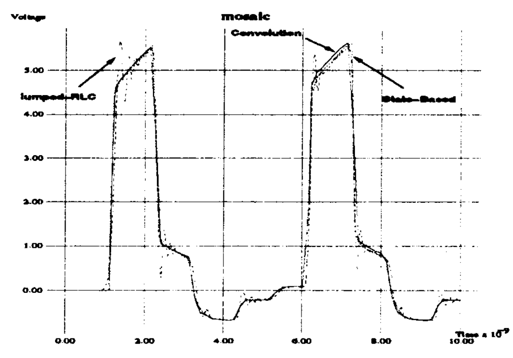

Figure 1: **mosaic** receiver-end voltage

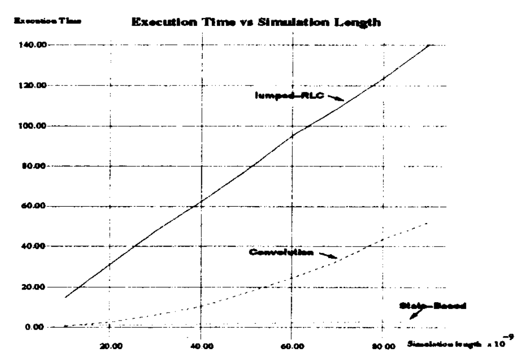

Figure 2: modified **mosaic**: execution-time vs simulation length

From Table 1 and Figure 2, it is observed that the state-based method can speed up simulations by factors of more than 10 and 50 over the convolution and lumped-RLC methods respectively.

[5] These circuits were provided by Raytheon Co.

Circuit	Simulation Length	Execution Time[a]		
		lumped-RLC	Convolution	State-Based
raytheon1	60 ns	739 s	19.43 s	20 s
	120 ns	1550 s	62.31 s	41.3 s
	180 ns	2237 s	131.32 s	60 s
	240 ns	3002 s	220s	78s
raytheon2	60 ns	336.35 s	37 s	28.7 s
	120 ns	668.3 s	110 s	52 s
	180 ns	1027 s	239 s	78.7 s
	240 ns	1372 s	380 s	99.6 s
	1000 ns	5646 s	6301 s	428 s
raytheon3	60 ns	885 s	40 s	28 s
	120 ns	1791 s	141 s	57 s
	180 ns	2700 s	529.41 s	95.3 s
mosaic	10 ns	44.44 s	0.9 s	2 s
	20 ns	93.18 s	3.6 s	2 s
	40 ns	181.3 s	12.9 s	3.5 s
	80 ns	371 s	49.5 s	6 s

[a] CPU times on a DEC 5000/200 running Ultrix 4.1

Table 1: Comparison of Execution Times

4 Conclusion

An accurate and efficient linear time-complexity method for lossy transmission line simulation has been presented. Experimental results indicate that the new technique can be significantly faster than other methods for long simulations. The method is currently being extended to apply to frequency-varying transmission line models[12].

Acknowledgments

Gerry Marino's encouragement and support of this work is greatly appreciated. Helpful technical insights were provided by Ken Kundert and Albert Ruehli. Thanks are due to Sally Liu and Richard Matick for their encouragement and to Beorn Johnson for useful discussions about implementation issues. The assistance of Sudeshna Sarkar and Berenice Ong was instrumental in the completion of this work. Administrative help from Kia Cooper is very much appreciated. This research was supported by AT&T Bell Laboratories, IBM, Raytheon and the California State MICRO program.

References

[1] C.A. Neugebauer et. al. High Performance Interconnections between VLSI Chips. *Solid State Tech.*, June 1988.

[2] J.E. Schutt-Aine and R. Mittra. Analysis of Pulse Propagation in Coupled Transmission Lines. *IEEE Trans. Ckts. Sys.*, CAS-32(12), December 1985.

[3] H.W. Dommel. Digital Computer Solution of Electromagnetic Transients in Single and Multiphase Networks. *IEEE Trans. Power App. Sys.*, PAS-88(4):388, April 1969.

[4] A.J. Gruodis. Transient Analysis of Uniform Resistive Transmission Lines in a Homogeneous Medium. *IBM J. Res. Dev.*, 23(6), November 1979.

[5] Omar Wing. Time-Domain Models of VLSI Interconnects. Talk at U.C. Berkeley, Apr. 1991.

[6] R. Wang and O. Wing. Analysis of VLSI Multiconductor Systems by Bi-Level Waveform Relaxation. In *Proc. ICCAD 90*, pages 166–169, 1990.

[7] A.R. Djordjević et. al. Analysis of Lossy Transmission Lines with Arbitrary Nonlinear Terminal Networks. *IEEE Trans. Microwave Th. Tech.*, MTT-34(6):660, June 1986.

[8] J.E. Schutt-Aine and R. Mittra. Nonlinear Transient Analysis of Coupled Transmission Lines. *IEEE Trans. Ckts. Sys.*, 36(7), July 1989.

[9] J.S. Roychowdhury and D.O. Pederson. Efficient Transient Simulation of Lossy Interconnect. Proc. 28th ACM/IEEE Design Automation Conference, June 17-21 1991, San Francisco.

[10] A. Deutsch et. al. High-speed signal propagation on lossy transmission lines. *IBM J. Res. Dev.*, 34(4):601–615, July 1990.

[11] J.S. Roychowdhury and A. R. Newton. Algorithms for the Transient Simulation of Lossy Interconnect. In preparation.

[12] J.S. Roychowdhury, A.R. Newton, and D.O. Pederson. State-Based Simulation of Frequency-Varying Lossy Line Models. To appear.