

# Cyclostationary Noise Analysis of Large RF Circuits with Multi-tone Excitations

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## Abstract

This paper introduces a new, efficient technique for analyzing RF noise in large circuits subjected to true multitone excitations. Noise statistics in such circuits are time-varying and are modelled in this work as cyclostationary stochastic processes, characterized in terms of their *harmonic power spectral densities* (HPSDs). Results from a large RF integrated circuit driven by an LO tone and a strong RF signal are presented. The analysis predicts correctly that the presence of the RF tone affects the noise significantly.

## 1 Introduction

Predicting noise is important in RF design. RF circuits are usually analyzed for their steady-state behaviour under one or more periodic excitations. Noise statistics in such circuits vary periodically or quasi-periodically in time, therefore SPICE-like stationary noise analyses do not capture important aspects such as frequency translation of noise spectra. In order to address such effects, noise needs to be modelled as cyclostationary stochastic processes. In this paper, a new RF noise formulation and algorithm, based on cyclostationary concepts, is presented.

Most previous algorithms [1, 2, 3] that model time-varying noise are limited to designs containing relatively few nonlinear elements, characteristic to microwave circuits. These methods are impractical for integrated RF circuits where nonlinear devices are numerous. Recently, an algorithm [4] was proposed that can analyze large circuits efficiently, but it is limited to single-tone excitations only. The technique presented in this paper can analyze large circuits with multitone large-signal inputs efficiently.

The statistics of cyclostationary processes are periodic or quasi-periodic, hence can be expressed as Fourier series. The technique of this work is formulated in terms of the coefficients of the Fourier series, termed *cyclostationary components*. The cyclostationary components, which the new algorithm computes efficiently, are useful in system-level analysis as equivalent noise models of RF circuit blocks. They also provide an intuitive yet mathematically rigorous visualization of RF noise propagation, which can contribute to design insight.

The algorithm is based on a structured block-matrix relation between the cyclostationary components of the noise variables within a circuit. It can compute the total noise at a specific output, correlations between noise at different outputs and also individual contributions from each noise generator to a specific output. Moreover, bias-dependent white and coloured noise sources (e.g., thermal, shot and flicker noises) are treated naturally, even when they are correlated. The algorithm has been verified against extensive Monte-Carlo noise simulations to an accuracy of 2%.

Cyclostationary processes are reviewed and the concept of harmonic power spectral densities is introduced in Section 2 through an example. The general noise formulation and the algorithm for large circuits are outlined in Section 3. In Section 4, the algorithm is illustrated on a large RF example.

## 2 Cyclostationary Noise and HPSDs

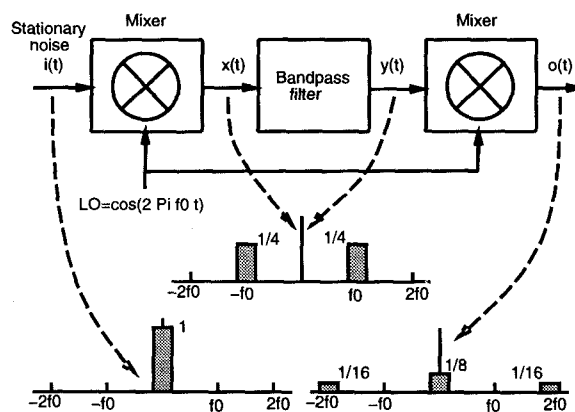


Figure 1: Mixer-filter-mixer circuit: naïve analysis

The circuit of Fig. 1 consists of a mixer, followed by a bandpass filter, followed by another mixer. This is a simplification of, e.g., the bias-dependent noise generation mechanism in semiconductor devices [5]. Both mixers multiply their inputs by a local oscillator of frequency  $f_0$ , i.e., by  $\cos(2\pi f_0 t)$ . The bandpass filter is centered around  $f_0$  and has a bandwidth of  $B \ll f_0$ . The circuit is noiseless, but the input to the first mixer is stationary band-limited noise with two-sided bandwidth  $B$ .

A naïve attempt to determine the output noise power would consist of the following analysis, illustrated in Fig. 1. The first mixer shifts the input noise spectrum by  $\pm f_0$  and scales it by  $1/4$ . The resulting spectrum is multiplied by the squared magnitude of the filter's transfer function. Since this spectrum falls within the pass-band of the filter, it is not modified. Finally, the second mixer shifts the spectrum again by  $\pm f_0$  and scales it by  $1/4$ , resulting in the spectrum with three components shown in the figure. The total noise power at the output, i.e., the area under the spectrum, is  $1/4$  that at the input.

This common but simplistic analysis is inconsistent with the following alternative argument. Note that the bandpass filter, which does not modify the spectrum of its input,

can be ignored. The input then passes through only the two successive mixers, resulting in the output noise voltage  $o(t) = i(t) \cos^2(2\pi f_0 t)$ . The output power is:

$$o^2(t) = i^2(t) \left[ \frac{3}{8} + \frac{\cos(2\pi 2f_0 t) + \cos(2\pi 4f_0 t)}{2} \right]$$

The average output power consists of only the  $3/8 i^2(t)$  term, since it can be shown that the cosine terms time-average to zero. Hence the average output power is  $3/8$  of the input power, 50% more than that predicted by the previous naïve analysis. This is, however, the correct result.

The contradiction between the above arguments underscores the need for a more rigorous analysis. Modelling circuit noises as stochastic processes provides the required generality and rigour. Since the local oscillator is periodic, the processes are *cyclostationary* [6, 7], i.e., their statistics vary periodically with time. The auto-correlation function of any cyclostationary process  $z(t)$  (defined as  $R_{zz}(t, \tau) = E[z(t)z(t + \tau)]$ ,  $E[\cdot]$  denoting expectation) can be expanded in a Fourier series in  $t$ :

$$R_{zz}(t, \tau) = \sum_{i=-\infty}^{\infty} R_{z_i}(\tau) e^{j i 2\pi f_0 t} \quad (1)$$

$R_{z_i}(\tau)$  are termed *harmonic autocorrelation functions*. The periodically time-varying power of  $z(t)$  is its autocorrelation function evaluated at  $\tau = 0$ , i.e.,  $R_{zz}(t, 0)$ . The quantities  $R_{z_i}(0)$  represent the harmonic components of the periodically-varying power. The average power is simply the value of the DC or *stationary component*,  $R_{z_0}(0)$ <sup>1</sup>. The frequency-domain representation of the harmonic autocorrelations are termed *harmonic power spectral densities* (HPSDs)  $S_{z_i}(f)$  of  $z(t)$ , defined as the Fourier transforms:

$$S_{z_i}(f) = \int_{-\infty}^{\infty} R_{z_i}(\tau) e^{-j\pi f \tau} d\tau \quad (2)$$

Equations can be derived that relate the HPSDs at the inputs and outputs of various circuit blocks. By solving these equations, any HPSDs in the circuit can be determined.

Consider, for example, the circuit in Fig. 1. The input and output HPSDs of a perfect cosine mixer with unit amplitude can be shown [8] to be related by:

$$S_{v_k}(f) = \frac{S_{u_{k-2}}(f - f_0)}{4} + \frac{S_{u_k}(f - f_0) + S_{u_k}(f + f_0)}{4} + \frac{S_{u_{k+2}}(f + f_0)}{4} \quad (3)$$

( $u$  and  $v$  denoting the input and output, respectively). The HPSD relation for a filter with transfer function  $H(f)$  is [8]:

$$S_{v_k}(f) = H(-f)H(f + kf_0) S_{u_k}(f) \quad (4)$$

The HPSDs of the circuit are illustrated in Fig. 2. Since the input noise  $i(t)$  is stationary, its only nonzero HPSD is the stationary component  $S_{i_0}(f)$ , assumed to be unity in the frequency band  $[-B/2, B/2]$ , as shown. From Equation 3 applied to the first mixer, *three* nonzero HPSDs ( $S_{x_0}$ ,  $S_{x_2}$  and  $S_{x_{-2}}$ , shown in the figure) are obtained for  $x(t)$ .

<sup>1</sup> *Stationary* processes are a special case of cyclostationary processes, where the autocorrelation function (hence the power) is independent of the time  $t$ ; it follows that  $R_{z_i}(\tau) \equiv 0$  if  $i \neq 0$ .

These are generated by shifting the input PSD by  $\pm f_0$  and scaling by  $1/4$ ; in contrast to the naïve analysis, the stationary HPSD is not the only spectrum used to describe the upconverted noise. From Equation 4, it is seen that the ideal bandpass filter propagates the three HPSDs of  $x(t)$  unchanged to  $y(t)$ . Through Equation 3, the second mixer generates five nonzero HPSDs, of which only the stationary component  $S_{o_0}(f)$  is shown in the figure. This is obtained by scaling and shifting not only the stationary HPSD of  $y(t)$ , but also the cyclostationary HPSDs, which in fact contribute an extra  $1/4$  to the lobe centered at zero. The average output noise (the shaded area under  $S_{o_0}(f)$ ) equals  $3/8$  of the input noise.

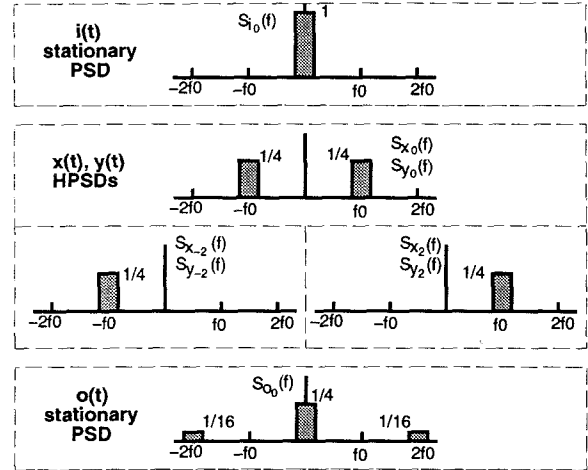


Figure 2: HPSDs of mixer-filter-mixer circuit

This simple example illustrates that HPSDs are a rigorous and convenient way to analyze RF noise. The HPSD formulation is a powerful one: incorporating a non-ideal filter is simple using Equation 4, and noise propagation through the circuit is easy to visualize. Non-intuitive results regarding the stationarity of filtered noise have also been established using HPSDs [8]. The formulation is useful not only for hand-calculations and proofs, but also for simulating large circuits, since the HPSDs of circuit unknowns obey a block-matrix relation. This equation, together with an efficient algorithm to compute it for large circuits, is described in Section 3.

### 3 Efficient cyclostationary noise computation algorithm

Noise in any nonlinear circuit can be analyzed using the circuit's ODEs:

$$\dot{q}(x(t)) + f(x(t)) + b(t) + Au(t) = 0 \quad (5)$$

$x(t)$  are the circuit unknowns,  $b(t)$  the periodic large-signal excitations and  $u(t)$  the device noise generators. Since the noise is small, it can be analyzed using a small-signal perturbation about the periodic noise-free solution  $x^*(t)$ . A time-varying linearization yields:

$$C(t) \dot{y} + G(t)y + Au(t) = 0 \quad (6)$$

where  $y(t)$  represents the small-signal deviations due to noise. Equation 6 describes a linear periodically time-varying

(LPTV) system with input  $u(t)$  and output  $y(t)$ . The system can be characterized by its time-varying transfer function  $H(t, f)$ .  $H(t, f)$  is periodic in  $t$  and can be expanded in a Fourier series similar to Equation 1. Denote the Fourier components (*harmonic transfer functions*) by  $H_i(f)$ .

Since  $u(t)$  and  $y(t)$  are vector processes, their autocorrelation functions are *matrices*  $R_{zz}(t, \tau) = E[z(t)z^T(t + \tau)]$ , consisting of auto- and cross-correlations. Similarly, the HPSDs  $S_{z_i}(f)$  are also matrices. One of the main contributions of this work is a block-matrix relation between the input and output HPSDs matrices (\* denotes the Hermitian):

$$S_{xx}(f) = \mathcal{H}(f) S_{uu}(f) \mathcal{H}^*(f) \quad (7)$$

$\mathcal{H}(f)$  (the *conversion matrix*) is the following block-structured matrix ( $f^k$  denotes  $f + kf_0$ ):

$$\mathcal{H}(f) = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \dots & H_0(f^1) & H_1(f^0) & H_2(f^{-1}) & \dots \\ \dots & H_{-1}(f^1) & H_0(f^0) & H_1(f^{-1}) & \dots \\ \dots & H_{-2}(f^1) & H_{-1}(f^0) & H_0(f^{-1}) & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \quad (8)$$

$S_{uu}(f)$  and  $S_{xx}(f)$  are similar to  $\mathcal{H}(f)$ : their transposes  $S_{zz}^T(f)$  have the same structure, but with  $H_i(f^k)$  replaced by  $S_{z_i}^T(f^k)$ .

Equation 7 expresses the output HPSDs, contained in  $S_{xx}(f)$ , in terms of the input HPSDs (contained in  $S_{uu}(f)$ ) and the harmonic transfer functions of the circuit (contained in  $\mathcal{H}(f)$ ). The HPSDs of a single output variable  $x_p(t)$  (both auto- and cross-terms with all other output variables) are available in the  $p^{\text{th}}$  column of the central block-column of  $S_{xx}^T(f)$ . To pick this column,  $S_{xx}^T(f)$  is applied to a unit vector  $E_{0p}$ , as follows ( $\bar{\cdot}$  denotes the conjugate):

$$S_{xx}^T(f) E_{0p} = \bar{\mathcal{H}}(f) S_{uu}^T(f) \mathcal{H}^T(f) E_{0p} \quad (9)$$

Evaluating Equation 9 involves two kinds of matrix-vector products:  $\mathcal{H}(f)z$  and  $S_{uu}(f)z$  for some vectors  $z$ . Consider the latter product first. If the inputs  $u(t)$  are stationary, as can be assumed without loss of generality [8], then  $S_{uu}(f)$  is block-diagonal. In practical circuits, the inputs  $u(t)$  are either uncorrelated or sparsely correlated. This results in each diagonal block of  $S_{uu}(f)$  being either diagonal or sparse. In both cases, the matrix-vector product can be performed efficiently.

The product with  $\mathcal{H}(f)$  can also be performed efficiently, by exploiting the relation  $\mathcal{H}(f) = J^{-1}(f)\mathcal{A}$  [9].  $\mathcal{A}$  is a sparse incidence matrix of the device noise generators, hence its product with a vector can be computed efficiently.  $J(0)$  is the harmonic balance Jacobian matrix [10] at the large-signal solution  $x^*(t)$ .  $J(f)$  is obtained by replacing  $kf_0$  by  $kf_0 + f$  in the expression for the Jacobian. By using iterative linear algebra techniques, it can be shown [10] that the product  $J^{-1}z$  can also be computed with effort that grows linearly with circuit size and almost linearly with the number of large-signal harmonics. As a result, Equation 9 can be computed efficiently for large circuits to provide the auto- and cross-HPSDs of any output of interest.

## 4 Results

The fast cyclostationary noise algorithm of Section 3 has been prototyped in a Bell Labs internal simulator. The

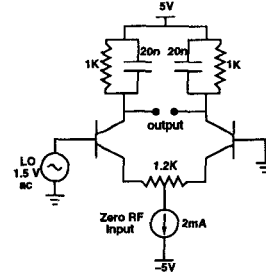


Figure 3: Mixer cell

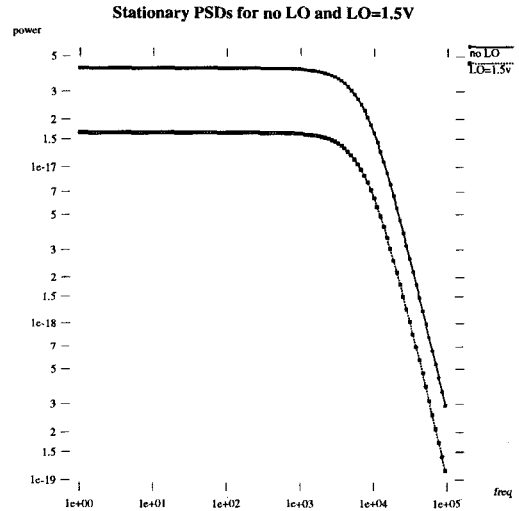


Figure 4: Mixer: Stationary PSD at output

algorithm has been verified against Monte-Carlo noise simulations with 60,000 sample waveforms, to an accuracy of within 2%. In this section, noise analysis results from two circuits are presented — a mixer excited by a single LO tone, and a large circuit, consisting of an I-channel buffer and mixer, driven by two strong tones (a signal and an LO).

### 4.1 Mixer analysis

The mixer in Fig. 3 was analyzed for cyclostationary noise to investigate the effect of large-signal LO variations on the output noise. The LO signal of amplitude 1.5V is applied at the base of the first transistor, as shown. The RF input signal is applied through the current source, which is held at a DC value of 2mA (i.e., no RF signal) for the noise analysis. Two simulations were performed: a stationary analysis with no LO present to obtain the noise of the quiescent circuit, and a cyclostationary analysis with the LO amplitude at 1.5V. The former simulation took a few seconds and the latter (with 25 large-signal harmonics) 40 seconds per frequency point. The stationary PSD is shown in Fig. 4, and some nonstationary HPSDs in Fig. 5.

From Fig. 4, it can be seen that the presence of a large LO signal reduces the average noise power at the output. This is a known property of switching mixers. Fig. 5 shows the first six harmonic PSDs of the noise at the output when the LO is 1.5V.

From a knowledge of the HPSDs, it is possible to cre-

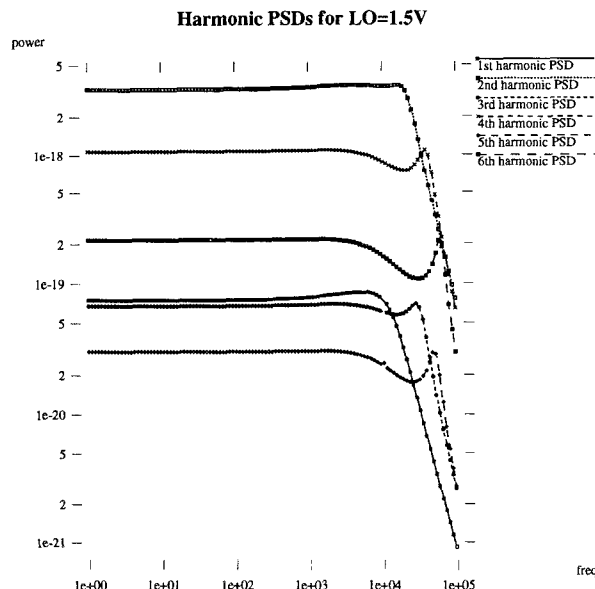


Figure 5: Mixer: Harmonic PSDs at output

ate system-level macromodels for functional blocks like the mixer. All the noise of the circuit can then be concentrated in an equivalent noise source with the same HPSDs. While only the stationary PSD determines the average noise power, the nonstationary HPSDs must be included because they can contribute to the stationary component of some other block, as discussed in Section 2.

## 4.2 I-channel buffer and mixer circuit

The next example is a portion of the W2013 RFIC, consisting of an I-channel buffer feeding a mixer. The circuit consisted of about 360 nodes, and was excited by two tones — a local oscillator at 178MHz driving the mixer, and a strong RF signal tone at 80Khz feeding into the I-channel buffer. Two noise analyses were performed. The first analysis included both LO and RF tones (sometimes called a three-tone noise analysis). The circuit was also analyzed with only the LO tone to determine if the RF signal affects the noise significantly. The two-tone noise simulation, using a total of 525 large-signal mix components, required 300MB of memory and for each frequency point, took 40 minutes on an SGI machine (200MHz R10000 CPU). The one-tone noise simulation, using 45 harmonics, needed 70MB of memory and took 2 minutes per point.

The stationary PSDs of the mixer output noise for the two simulations are shown in Fig. 6. It can be seen that the presence of the large RF signal increases the noise by about 1/3. This effect is difficult to predict with the technique of [4]. The peaks in the two waveforms, located at the LO frequency, are due to noise shifted from other frequencies.

## 5 Conclusion

An efficient frequency-domain algorithm has been presented for computing noise in nonlinear circuits. The method uses harmonic PSDs in its noise formulation. A block-structured matrix equation for the output noise statistics is the central

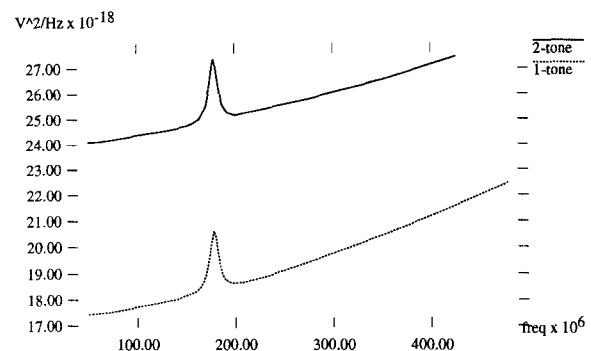


Figure 6: Stationary PSDs for the I-Q mixer/buffer circuit

result enabling the fast algorithm. The algorithm, which has been verified against Monte-Carlo simulations, is efficient for circuits with many large tones and can generate information useful for noise macromodels. Results from a mixer cell and a large I-channel buffer and mixer RF integrated circuit have been presented, predicting the fact that the presence of multiple tones can significantly affect the noise performance of a circuit.

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