

Phase Noise and Timing Jitter in Oscillators

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Abstract

Phase noise is a topic of theoretical and practical interest in electronic circuits, as well as in other fields such as optics. Although progress has been made in understanding the phenomenon, there still remain significant gaps, both in its fundamental theory and in numerical techniques for its characterisation. We present a **rigorous nonlinear analysis** for phase noise in oscillators and reach the following conclusions:

- The power spectrum of an oscillator **does not blow up** at the carrier frequency as predicted by many previous analyses. Instead, the shape of the spectrum is a **Lorentzian** (the shape of the squared magnitude of a one-pole lowpass filter transfer function) about each harmonic.
- The average spread (variance) of the timing jitter grows exactly linearly with time. A **single scalar constant** suffices to characterise both the timing jitter and spectral broadening due to phase noise.
- Previous linear analyses of phase noise make unphysical predictions such as infinite noise power.

We develop **efficient computational methods** in the time and frequency domains for predicting phase noise. Our techniques are practical for large circuits. We obtain good matches between spectra predicted using our technique and measured results, even at frequencies close to the carrier and its harmonics, where most previous techniques break down.

1 Introduction

Oscillators are ubiquitous in physical systems, especially electronic and optical ones. For example, in radio frequency (RF) communication systems, they are used for frequency translation of information signals and for channel selection. Oscillators are also present in digital electronic systems which require a time reference, i.e., a clock signal, in order to synchronise operations.

Noise is of major concern in oscillators, because introducing even small noise into an oscillator leads to dramatic changes in its frequency spectrum and timing properties. This phenomenon, peculiar to oscillators, is known as *phase noise* or *timing jitter*. A perfect oscillator would have localized tones at discrete frequencies (i.e., harmonics), but any corrupting noise spreads these perfect tones, resulting in high power levels at neighbouring frequencies. This effect is the major contributor to undesired phenomena such as interchannel interference, leading to increased bit-error-rates (BER) in RF communication systems. Another manifestation of the same phenomenon, jitter, is important in clocked and sampled-data systems: uncertainties in switching instants caused by noise lead to synchronisation problems. Characterising how noise affects oscillators is therefore crucial for practical applications. The problem is challenging, since oscillators constitute a special class among noisy physical systems: their *autonomous* nature makes them unique in their response to perturbations.

Considerable effort has been expended over the years in understanding phase noise and in developing analytical, computational and experimental techniques for its characterisation (see Section 2 for a brief review). Despite the importance of the problem and the large number of publications on the subject, a consistent and general treatment, and computational techniques based on a sound theory, appear to be still lacking. In this work, we provide a novel, rigorous theory for phase noise which leads to efficient numerical methods for its characterisation. Our techniques and results are general; they are applicable to *any* oscillatory system, electrical (resonant, ring, relaxation, etc.) or otherwise (gravitational, optical, mechanical, biological, etc.). We apply our numerical techniques to a variety of practical oscillator designs and obtain good matches against measurements.

The paper is organised as follows. In Section 2, we give a brief review of the previous work, and in Section 3, we present an overview

of our main results. Because of space limitations, discussion of background material, proofs and derivations [DMR97b] are omitted. In Section 4, we derive several quantities commonly used in oscillator design to quantify jitter and spectral properties, and in Section 5, we apply our methods to several practical electrical oscillators.

2 Previous work

A great deal of literature is available on the phase noise problem. Here we mention only some selected works from the fields of electronics and optics. Most investigations of electronic oscillators aim to provide insight into frequency-domain properties of phase noise, in order to develop rules for designing practical oscillators; well-known references include [Lee66, Rob91, Roh83, Vig94, Raz95]. Usually, these approaches apply linear time-invariant (LTI) analysis to high-Q or quartz-crystal type oscillators designed using standard feedback topologies. While often of great practical importance, such analyses often require large simplifications of the problem, and skirt fundamental issues such as why noisy oscillators exhibit spectral dispersion whereas forced systems do not. Attempts to improve on LTI analysis have borrowed from linear time-varying (LTV) analysis methods for forced (nonoscillatory) systems (e.g., [Haf66, Kur68, RCMC94, OT97]). LTV analyses can predict spectra more accurately than LTI ones in some frequency ranges; however, LTV techniques for forced systems retain nonphysical artifacts of LTI analysis (such as infinite output power) and provide no real insight into the basic mechanism generating phase noise.

Oscillators that rely on abruptly switching elements, e.g., ring and relaxation oscillators, are more amenable to noise analysis. Perturbations cause variations in element delays, or alter the time taken to reach switching thresholds, thus directly determining timing jitter. [WKG94, McN94, AM83] predict phase noise by using analytical techniques on idealized models of specific oscillator circuits. The mechanism of such oscillators suggests the fundamental intuition that timing or phase errors increase with time. However, this intuition does not extend naturally to other types of oscillators.

More sophisticated analysis techniques predominate in the domain of optics [Lax67]. Here, stochastic analysis is common, and it is well known that phase noise due to white noise perturbations is described by a random walk process. Although justifications of this fact are often based on approximations, precise descriptions of phase noise have been obtained for certain systems. The fact that a random walk phase noise process leads to Lorentzian power spectra is also well established, e.g., [Fos88, VV83]. However, a general theory is apparently not available even in this field.

Possibly the most general and rigorous treatment of phase noise to date has been that of Kärtner [Kär90]. In this work, the oscillator response is decomposed into phase and magnitude components, and a differential equation is obtained for phase error. By solving a linear, small-time approximation to this equation with stochastic inputs, Kärtner obtains the correct Lorentzian spectrum for the power spectral density due to phase noise. Despite these advances, certain gaps remain, particularly with respect to the derivation and solution of the linear differential equation for phase error.

Recently, Hajimiri [HL97] has proposed a phase noise analysis based on a conjecture for decomposing perturbations into two (orthogonal) components, generating purely phase and amplitude deviations respectively. This intuition is similar to Kärtner's approach [Kär90]. Unfortunately, the conjecture for orthogonally decomposing the perturbation into components that generate phase and amplitude deviations, while intuitively appealing, can be shown to be invalid [DMR97a]. Design intuition resulting from the conjecture about noise source contributions can also be misleading.

In summary, the available literature often identifies basic and useful

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facets of phase noise separately, but lacks a unifying theory clarifying its fundamental mechanism. Furthermore, existing numerical methods for phase noise are based on forced-system concepts which are inappropriate for oscillators and can generate incorrect predictions.

3 Overview of main results

Consider the oscillator shown in Figure 1, consisting of a lossy LC circuit with an amplitude-dependent gain provided by the nonlinear resistor. The nonlinear resistor has a negative resistance region which pumps energy into the circuit when the capacitor voltage drops, thus maintaining stable oscillation. A current source $b(t)$ is also present, representing external perturbations due to noise. When there is no perturbation, i.e., $b(t)$ is zero, the oscillator oscillates with a perfectly periodic signal $x_s(t)$ (a vector consisting of the capacitor voltage and the inductor current), shown in Figure 2(a). In the frequency domain, the unperturbed waveform consists of a series of impulses at the fundamental and harmonics of the time period, as shown in Figure 2(b).

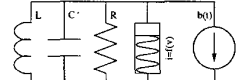
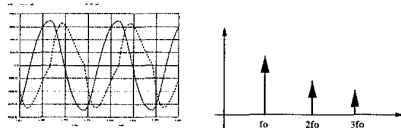


Figure 1: Simple oscillator



(a) Time domain (b) Frequency domain

Figure 2: Oscillator waveforms

In general, the dynamics of any oscillator can be described by a system of differential equations:

$$\dot{x} = f(x) \quad (1)$$

where $x \in \mathbb{R}^n$ and $f(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^n$. We consider systems that have a periodic solution $x_s(t)$ (with period T) to (1), i.e., a stable limit cycle in the n -dimensional solution space. We are interested in the response of such systems to a small state-dependent perturbation of the form $B(x)b(t)$ where $B(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times p}$ and $b(\cdot) : \mathbb{R} \rightarrow \mathbb{R}^p$. Hence the perturbed system is described by

$$\dot{x} = f(x) + B(x)b(t) \quad (2)$$

Although our eventual intent is to understand the response of the oscillator when $b(t)$ is random noise, it is useful to consider first the case when $b(t)$ is a known deterministic signal. We carry out a rigorous analysis of this case and obtain the following results:

1. The unperturbed oscillator's periodic response $x_s(t)$ is modified to $x_s(t + \alpha(t)) + y(t)$ by the perturbation, where:
 - (a) $\alpha(t)$ is a changing time shift, or *phase deviation*, in the periodic output of the unperturbed oscillator.
 - (b) $y(t)$ is an additive component, which we term the *orbital deviation*, to the phase-shifted oscillator waveform.
2. $\alpha(t)$ and $y(t)$ can always be chosen such that:
 - (a) $\alpha(t)$ will, in general, keep increasing with time even if the perturbation $b(t)$ is always small.
 - (b) the orbital deviation $y(t)$, on the other hand, will always remain small.

These results concretise existing intuition amongst designers about oscillator operation. Our proof of these facts is mathematically rigorous; further, we derive equations for $\alpha(t)$ and $y(t)$ which lead to *qualitatively different results* about phase noise compared to previous attempts. This is because our results are based on a new *nonlinear perturbation analysis* that is valid for oscillators, in contrast to previous approaches that rely on linearisation. Analysis based on linearisation

is not consistent for oscillators and results in non-physical predictions. Our approach leads to a *nonlinear differential equation* for the phase shift $\alpha(t)$:

$$\frac{d\alpha(t)}{dt} = v_1^T(t + \alpha(t))B(x_s(t + \alpha(t)))b(t), \quad \alpha(0) = 0 \quad (3)$$

where $v_1(t)$ is periodically time-varying vector, which we call the *Floquet vector* [DMR97b]. The Floquet vector, which can be computed efficiently, plays a crucial role in our analysis. Because of space limitations, we will not provide the derivation of (3) here. For the derivations and proofs of the above results, we use the Floquet theory [Far94] of linear periodically time-varying systems. From (3), one can see that if the perturbation is *orthogonal* to the Floquet vector for all t , the phase error $\alpha(t)$ is zero. The Floquet vector, in general, has no relationship to the tangent vector $\dot{x}_s(t)$ to the limit cycle. Authors in [HL97] conjecture that if the perturbation is orthogonal to the tangent vector, then there is no phase error. This conjecture, even though it might sound intuitive, is not correct. The direction of the perturbation that results in zero phase error is the direction that is orthogonal to the Floquet vector, which is in general (and for almost all practical oscillators) *oblique* to the tangent.

Next, we consider the case where the perturbation $b(t)$ is random noise, i.e., a vector of white noise processes – this situation is important for determining practical figures of merit like zero-crossing jitter and spectral purity (i.e., spreading of the power spectrum). Jitter and spectral spreading are in fact closely related, and both are determined by the manner in which $\alpha(t)$, now also a random process, spreads with time. We consider random perturbations in detail and establish that:

1. The average spread of the jitter (mean-square jitter, or variance) increases precisely *linearly* with time, i.e.,

$$\mathbb{E} [\alpha^2(t)] = \sigma^2(t) = ct \quad (4)$$

where

$$c = \frac{1}{T} \int_0^T v_1^T(\tau)B(x_s(\tau))B^T(x_s(\tau))v_1(\tau)d\tau \quad (5)$$

2. The power spectrum of the perturbed oscillator is a *Lorentzian* about each harmonic. A Lorentzian is the shape of the squared magnitude of a one-pole lowpass filter transfer function.
3. A *single scalar constant* c is sufficient to describe jitter and spectral spreading in a noisy oscillator.
4. The oscillator's output with phase noise, i.e., $x_s(t + \alpha(t))$ is a *stationary* stochastic process.

If we define X_i to be the Fourier coefficients of $x_s(t)$:

$$x_s(t) = \sum_{i=-\infty}^{\infty} X_i \exp(ji2\pi f_0 t)$$

then, the spectrum of the stationary oscillator output $x_s(t + \alpha(t))$ is given by

$$S(f) = 2 \sum_{i=-\infty}^{\infty} X_i X_i^* \frac{f_0^2 i^2 c}{\pi^2 f_0^4 i^4 c^2 + (f + i f_0)^2} \quad (6)$$

where $f_0 = 1/T$ is the fundamental frequency. The phase deviation $\alpha(t)$ does *not* change the total power in the periodic signal $x_s(t)$, but it alters the power density in frequency, i.e., the power spectral density. For the perfect periodic signal $x_s(t)$, the power spectral density has δ functions located at discrete frequencies (i.e., the harmonics). The phase deviation $\alpha(t)$ *spreads* the power in these δ functions in the form given in (6), which can be experimentally observed with a spectrum analyzer.

The above results have important implications. The Lorentzian shape of the spectrum implies that the power spectral density at the carrier frequency and its harmonics has a finite value, and that the total carrier power is preserved despite spectral spreading due to noise. Previous analyses based on linear time-invariant (LTI) or linear time-varying (LTV) concepts erroneously predict infinite noise power density at the carrier, as well as infinite total integrated power. That the

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oscillator output is stationary is surprising at first sight, since oscillators are nonlinear systems with periodic swings, hence it might be expected that output noise power would change periodically as in forced systems. However, it must be remembered that while forced systems are supplied with an external time reference (through the forcing), oscillators are not. Cyclostationarity in the oscillator's output would, by definition, imply a time reference. Hence the stationarity result reflects the fundamental fact that noisy autonomous systems cannot provide a perfect time reference.

We apply our theory above to develop correct (both time and frequency domain) computational techniques that are efficient for practical circuits [DMR97b]. We derive new numerical methods for jitter/spectral dispersion, with the following features:

1. The methods require only a knowledge of the steady state of the unperturbed oscillator, and the values of the noise generators.
2. Large circuits are handled efficiently, i.e., computation/memory scale linearly with circuit size.
3. The separate contributions of noise sources, and the sensitivity of phase noise to individual circuit devices and nodes, can be obtained easily.

We use our theory and numerical methods to analyse a variety of oscillators, and compare the results against measurements. We obtain good matches even at frequencies close to the carrier, unlike most previous analyses. Our numerical methods are more than three orders of magnitude faster than Monte-Carlo simulations, the only alternative method for producing qualitatively correct predictions. The brute-force Monte-Carlo technique is the only previously available analysis method, apart from ours, that is not based on linearisation.

State space interpretations of phase and orbital deviation

The phase and orbital deviations have intuitive interpretations when the oscillator's response is viewed in the state-space or phase plane. In Figure 3, the voltage $v(t)$ across the capacitor is plotted against the current $i(t)$ through the inductor. The trace for the unperturbed oscillator forms a closed curve since this waveform is perfectly periodic. When the oscillator is perturbed, this periodicity is lost. *For stable oscillators, however, the perturbed trajectory remains within a small band around the unperturbed trajectory*, as shown.

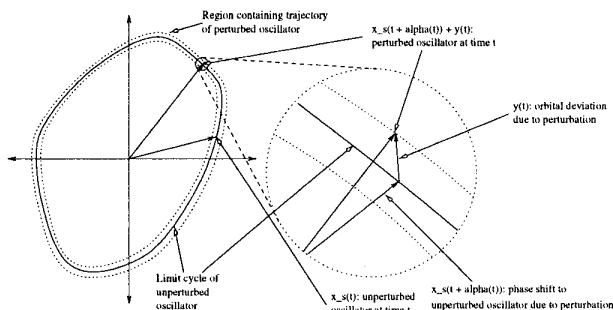


Figure 3: Oscillator trajectories

The closeness of the perturbed and unperturbed trajectories in the phase plane does not imply that the time-domain waveforms are also close to each other. The points on the perturbed and unperturbed trajectory corresponding to a given time t will, in general, be far from each other, as illustrated in Figure 3. However, the waveform of the perturbed oscillator does remain close to the unperturbed waveform after it is time-shifted by $\alpha(t)$. In the figure, this time or phase shift results in the difference between the unperturbed point $x_s(t)$ and the "phase component" $x_s(t + \alpha(t))$ of the perturbed trajectory. The orbital deviation $y(t)$ due to the perturbation is also shown.

$\alpha(t)$ grows very much like the integral of the perturbation. For a constant perturbation, for example, $\alpha(t)$ is approximately a linear ramp. This indicates how the frequency of the oscillator can change due to perturbations, for a linearly increasing phase error is equivalent to a frequency error. It also suggests why cycle-to-cycle (i.e., per cycle) timing jitter is a constant quantity.

4 Phase noise/timing jitter characterisation

In this section, we discuss several popular characterisations of phase noise that is used in the design of electronic oscillators, and how they can easily be obtained from the stochastic characterisation we described in Section 3.

Single-sideband phase noise spectrum in dBc/Hz

In practice, we are usually interested in the PSD around the first harmonic, i.e., $S(f)$ for f around f_0 . The *single-sideband* phase noise $\mathcal{L}(f_m)$ (in dBc/Hz) that is very widely used in practice is defined as

$$\mathcal{L}(f_m) = 10 \log_{10} \left(\frac{S(f_0 + f_m)}{2 |X_1|^2} \right) \quad (7)$$

For "small" values of c , and for $0 \leq f_m \ll f_0$, (7) can be approximated as

$$\mathcal{L}(f_m) \approx 10 \log_{10} \left(\frac{f_0^2 c}{\pi^2 f_0^4 c^2 + f_m^2} \right) \quad (8)$$

Furthermore, for $\pi f_0^2 c \ll f_m \ll f_0$, $\mathcal{L}(f_m)$ can be approximated by

$$\mathcal{L}(f_m) \approx 10 \log_{10} \left(\left(\frac{f_0}{f_m} \right)^2 c \right) \quad (9)$$

Notice that the approximation of $\mathcal{L}(f_m)$ in (9) blows up as $f_m \rightarrow 0$. For $0 \leq f_m < \pi f_0^2 c$, (9) is not accurate, in which case the approximation in (8) should be used.

Timing jitter

In some applications, such as clock generation and recovery, one is interested in a characterisation of the phase/time deviation $\alpha(t)$ itself rather than the spectrum of $x_s(t + \alpha(t))$. In these applications, an oscillator generates a square-wave like waveform to be used as a clock. The effect of the phase deviation $\alpha(t)$ on such a waveform is to create jitter in the *zero-crossing* or *transition* times. In Section 3, we stated that $\alpha(t)$ (for an autonomous oscillator) becomes a random variable with a linearly increasing variance, i.e., $\sigma^2(t) = ct$. Let us take one of the transitions (i.e., edges) of a clock signal as a reference (i.e., trigger) transition and synchronize it with $t = 0$. If the clock signal is perfectly periodic, then one will see transitions exactly at $t_k = kT$, $k = 1, 2, \dots$ where T is the period. For a clock signal with a phase deviation $\alpha(t)$ that has a linearly increasing variance as above, the timing of the k th transition t_k will have a variance (i.e., mean-square error)

$$\mathbb{E} [(t_k - kT)^2] = c k T \quad (10)$$

The spectral dispersion caused by $\alpha(t)$ in an oscillation signal can be observed with a spectrum analyzer. Similarly, one can observe the timing jitter caused by $\alpha(t)$ using a sampling oscilloscope. McNeill in [McN94] experimentally observed the linearly increasing variance for the timing of the transitions of a clock signal generated by an autonomous oscillator, as predicted by our theory. Moreover, c (in $\text{sec}^2 \cdot \text{Hz}$) in (10) exactly quantifies the rate of increase of timing jitter with respect to a reference transition. Another useful figure of merit is the *cycle-to-cycle* timing jitter, i.e., the timing jitter in one clock cycle, which has a variance cT .

Noise source contributions

The scalar constant c appears in the characterisations we discussed above. It is given by (5), where $B(\cdot) : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times p}$ represents the modulation of the intensities of the noise sources with the large-signal state. (5) can be rewritten as

$$c = \sum_{i=1}^p \frac{1}{T} \int_0^T [v_i^T(\tau) B_i(\tau)]^2 d\tau = \sum_{i=1}^p c_i \quad (11)$$

where p is the number of the noise sources, i.e., the column dimension of $B(x_s(\cdot))$, and $B_i(\cdot)$ is the i th column of $B(x_s(\cdot))$ which maps the i th noise source to the equations of the system. Hence,

$$c_i = \frac{1}{T} \int_0^T [v_i^T(\tau) B_i(\tau)]^2 d\tau \quad (12)$$

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represents the contribution of the i th noise source to c . Thus, the ratio

$$\frac{c_i}{c} = \frac{c_i}{\sum_{i=1}^p c_i} \quad (13)$$

can be used as a *figure of merit* representing the contribution of the i th noise source to phase noise/timing jitter. Note that the phase error $\alpha(t)$ is described by a nonlinear differential equation where the noise sources are the excitations. Hence, one can not use the superposition principle to calculate the phase error arising from multiple noise sources. On the other hand, the phase error variance $\sigma^2(t) = ct$ is linearly related to the noise sources, i.e., the variance of phase error due to two noise sources is the summation of the variances due to the noise sources considered separately.

Phase noise sensitivity

One can define

$$c_s^{(k)} = \frac{1}{T} \int_0^T [v_1^T(\tau) e_k]^2 d\tau \quad (14)$$

(where $1 \leq k \leq n$ and e_k is the k th unit vector) as the *phase noise/timing jitter sensitivity* of the k th equation (i.e., node), because e_k represents a unit intensity noise source added to the k th equation (i.e., connected to the k th node) in (1). The phase noise sensitivity of nodes can provide useful information in search for novel oscillator architectures with low phase noise.

5 Examples

We now present examples for phase noise characterisation of practical electronic oscillators.

Oscillator with a bandpass filter and a nonlinearity [DTS97]

This oscillator (Figure 5) consists of a Tow-Thomas second-order bandpass filter and a comparator [DTS97]. If the OpAmps are considered to be ideal, it can be shown that this oscillator is equivalent (in the sense of the differential equations that describe it) to a parallel RLC circuit in parallel with a nonlinear voltage-controlled current source (or equivalently a series RLC circuit in series with a nonlinear current-controlled voltage source) as in Figure 1. In [DTS97], authors breadboarded this circuit with an external white noise source (intensity of which was chosen such that its effect is much larger than the other internal noise sources), and measured the PSD of the output with a spectrum analyzer. For $Q = 1$ and $f_0 = 6.66$ kHz, we performed a phase noise characterisation of this oscillator using our numerical methods, and computed the periodic oscillation waveform $x_s(t)$ for the output and $c = 7.56 \times 10^{-8} \text{ sec}^2 \cdot \text{Hz}$. Figure 5(a) shows the PSD of the oscillator output computed using (6), and Figure 5(b) shows the spectrum analyzer measurement¹. The single-sideband phase noise spectrum using both (8) and (9) is in Figure 5. Note that (9) can not predict the PSD accurately below the cut-off frequency $f_c = \pi f_0^2 c = 10.56$ Hz (marked with a * in Figure 5) of the Lorentzian.

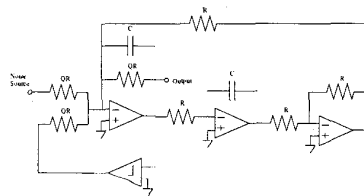
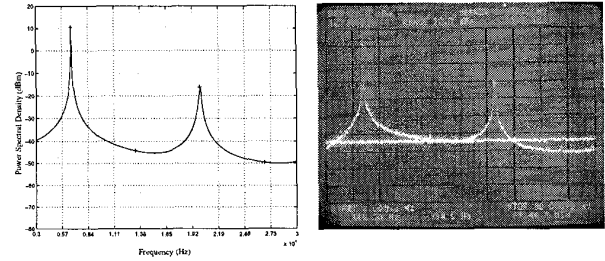


Figure 4: Band-pass filter and a comparator

2.5 GHz Oscillator with on-chip inductor [Kin97]

A simplified schematic for this oscillator is in Figure 7. We computed $c = 1.34 \times 10^{-19} \text{ sec}^2 \cdot \text{Hz}$ which corresponds to $\mathcal{L}(f_m) = -100.65 \text{ dBc/Hz}$ at $f_m = 100 \text{ KHz}$ using (9). The measured SSB phase noise at $f_m = 100 \text{ KHz}$ is $\mathcal{L}(f_m) = -96 \text{ dBc/Hz}$. We believe that the 4.65 dBc/Hz difference between the measured result and the one simulated is due to the following: (a) The oscillator circuit simulated was not extracted from the layout. (b) The MOS device models and the noise models used in the simulation were not characterised for

¹The PSDs are plotted in units of dBm.



(a) Computed PSD (4 harmonics) (b) Measured PSD [DTS97]

Figure 5: Computed and measured PSD

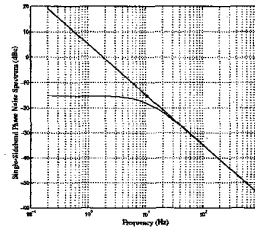


Figure 6: $\mathcal{L}(f_m)$ computed with both (8) and (9)

the process the oscillator was fabricated with. We generated a noise source contribution report for this oscillator using (13). M1, M2 contribute 41% to c , M3, M4 and M5 contribute 15%, 35% is from inductor losses, and 9% is from the rest of the noise sources in the circuit.

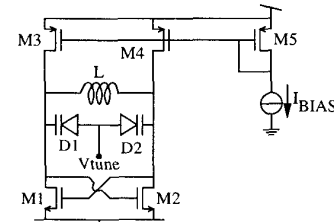


Figure 7: Oscillator with on-chip inductor: simplified schematic

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