MPDE Methods for Efficient Analysis of Wireless Systems

Jaijeet Roychowdhury
jaijeet@research.bell-labs.com
Bell Laboratories, Murray Hill

Abstract

Recently, the MPDE (multirate partial differential equation) formulation was introduced for analysing circuits with widely-separated time scales [Roy98]. A striking feature of the MPDE is that it leads to efficient methods for simulating several categories of circuits that are difficult or impossible to analyse by existing simulation techniques. In this paper, numerical methods based on the MPDE are applied to several subsystems important in portable wireless systems, namely *DC-DC power converters, switched-capacitor filters* and *RF mixers*. Simulations indicate speedups of more than two orders of magnitude over traditional methods. It is also shown how the MPDE can be used to abstract compact high-level models from detailed circuit descriptions of wireless system components.

1 Introduction

The presence of widely-separated time scales in circuits such as power converters, switched-capacitor circuits, RF mixers, etc., creates a significant bottleneck for existing simulation tools. Such circuits combine strongly nonlinear elements with at least two well-separated time scales (e.g., clock/LO and signal), and this combination of properties often breaks traditional numerical methods (e.g., transient analysis, shooting, harmonic balance). Available tools are typically slow and inaccurate when dealing with such circuits.

Recently, we introduced a general formulation for multi-rate circuits [Roy98, Roy97] called the *multi-rate partial differential equation (MPDE)*. By representing multi-rate signals as functions of more than one time variable (see Section 3 for an illustration), this formulation makes it possible to analyse strong nonlinearities together with widely-separated time scales efficiently. In this paper, we apply new MPDE-based numerical techniques to circuits for *power-conversion, switched-capacitor filtering* and *switched RF mixing*. These circuits, important in wireless/portable applications, often take disproportionately long to design because of the lack of effective simulation methods. The new methods are more than two orders of magnitude faster than traditional techniques and also produce more accurate results.

We also show how the MPDE addresses the problem of generating accurate higher-level models of complex building blocks. This capability is useful for bottom-up design of wireless and other systems. MPDE-based analysis is used to generate representations of linear time-varying (LTV) transfer functions, which often describe the operation of communication systems adequately. Reduced-order modelling techniques for linear operators are used to produce smaller, higher-level models that are cheaper to evaluate, yet approximate the input-output behaviour of the more complex system well. Capturing the time-varying nature of these transfer functions ensures that important phenomena like frequency translation are modelled correctly.

In Section 2, previous simulation techniques relevant to the multi-rate problem are reviewed briefly. The MPDE formulation, together with new numerical methods based on it, is described in Section 3. MPDE-based reduction of LTV transfer functions is presented in Section 4. Application of the new methods to several circuits is described in Section 5.

2 Previous methods and limitations

Existing time-domain methods such as transient integration and shooting, while suitable for strong nonlinearities, have difficulty with widely separated time-scales. They are forced to follow the details of waveforms at the fastest time scale, for the much larger durations of the slower time scales. This results in an excessive number of simulation timepoints and also accuracy loss.

Frequency-domain methods like harmonic balance (e.g., [KWSV90]) can, on the other hand, handle several time scales or tones, but have difficulty with strong nonlinearities. The reason is that strong nonlinearities tend to generate signals with sharp edges and corners, which are inefficient to represent in the frequency domain because they require many Fourier components. Strong nonlinearities also cause problems with preconditioning, needed in recent harmonic balance methods (e.g., [MFR95]).

Mixed frequency-time methods, such as those of Ushida and Chua [UC84] and Kundert et al [KWSV90], have attempted to combine the advantages of both domains. However, these techiques rely on highly localized sampling, resulting in numerical ill-conditioning. They are also limited to only one strongly nonlinear tone. The specialized program SWITCAP [FTW83], designed for switched-capacitor circuits, uses idealized switch models to achieve significant speedups over more general algorithms. This approach ignores important nonlinear effects; in particular, it is limited in its capability to predict signal path harmonic distortion, a critical figure of merit for SC designs.

3 MPDE-based numerical algorithms

In this section, an overview of the MPDE and related numerical techniques is presented. Many details are omitted for brevity and may be found in [Roy98].

The key to the MPDE formulation is the use of multivariate functions (functions of several time variables) to represent signals with separated time scales efficiently. To understand the concept, consider the product of a 1 Hz sine wave and a 1Ghz pulse train, given by:

$$y(t) = \sin(2\pi t)\ \text{pulse}\left(\frac{t}{10^9}\right) \quad (1)$$

Figure 1 depicts $y(t)$, with the pulse train frequency of 10^9 changed to 50 for viewing convenience. This quasi-periodic signal is expensive to represent in the time domain because 10^9 pulses of different shapes need to be sampled before the waveform repeats. It is this problem that makes traditional time-domain techniques like SPICE's transient analysis inefficient for such signals. Representation in the frequency domain as a two-tone signal is also inefficient because the pulses require many Fourier components for accuracy.

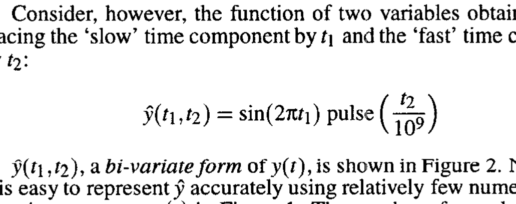

Figure 1: $y(t)$

Consider, however, the function of two variables obtained by replacing the 'slow' time component by t_1 and the 'fast' time component by t_2:

$$\hat{y}(t_1,t_2) = \sin(2\pi t_1)\ \text{pulse}\left(\frac{t_2}{10^9}\right) \quad (2)$$

$\hat{y}(t_1,t_2)$, a *bi-variate form* of $y(t)$, is shown in Figure 2. Notice that it is easy to represent $\hat{y}$ accurately using relatively few numerical samples, in contrast to $y(t)$ in Figure 1. The number of samples does not depend on the separation of the two time scales, which merely determines the scaling of the axes. Moreover, $y(t)$ can be easily obtained by

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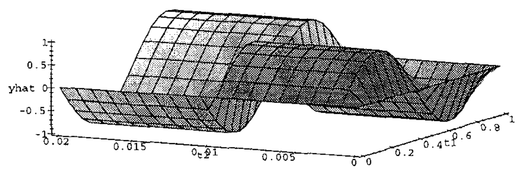


Figure 2: $\hat{y}(t_1,t_2)$

interpolation from samples of $\hat{y}(t_1,t_2)$, using the fact that $y(t)=\hat{y}(t,t)$ and that $\hat{y}(t_1,t_2)$ is periodic in each argument.

This observation is the basis of the MPDE formulation, in which all the waveforms in a circuit are represented in their bi-variate forms (or multivariate forms if there are more than two time scales). The key to efficiency is to solve for these waveforms directly, without involving the numerically inefficient one-dimensional forms at any point. To do this, it is necessary to first describe the circuit's equations using the multivariate functions. The traditional form of a circuit's equations, used in all simulators, is the Differential-Algebraic Equation (DAE):

$$\dot{q}(x)+f(x)=b(t) \tag{3}$$

$x(t)$ is the vector of circuit unknowns (node voltages and branch currents); q denotes the charge/flux terms and f the resistive terms; $b(t)$ is the vector of excitations to the circuit (typically from independent voltage/current sources). It can be shown [Roy98] that if $\hat{x}(t_1,t_2)$ and $\hat{b}(t_1,t_2)$ denote the bi-variate forms of the circuit unknowns and excitations, then the following MPDE is the correct generalization of (3) to the bi-variate case:

$$\boxed{\frac{\partial q(\hat{x})}{\partial t_1}+\frac{\partial q(\hat{x})}{\partial t_2}+f(\hat{x})=\hat{b}(t_1,t_2)} \tag{4}$$

More precisely, if $\hat{b}$ is chosen to satisfy $b(t)=\hat{b}(t,t)$, and $\hat{x}$ satisfies (4), then it can be shown that $x(t)=\hat{x}(t,t)$ satisfies (3). Also, if (3) has a quasi-periodic solution, then (4) can be shown to have a corresponding bi-variate solution.

By solving the MPDE numerically in the time domain, strong nonlinearities can be handled efficiently. The following new methods have been developed for solving (4):

1. **Quasi-periodic time-domain methods (MFDTD and HS):** Quasi-periodic solutions are found by enforcing bi-periodic boundary conditions on the MPDE. In the Multivariate Finite Difference Time Domain (MFDTD), (4) is discretized on a grid in the t_1-t_2 plane by approximating the differentiation operators with a numerical differentiation formula. The resultant system of nonlinear equations, together with the bi-periodic boundary conditions, is solved using a nonlinear solution method. The grid is refined adaptively so that the solution is captured efficiently. Another purely time-domain method, Hierarchical Shooting (HS), is a generalization of the traditional shooting method to multiple time scales. Both MFDTD and HS are appropriate for signals with strongly nonlinear activity in every component, such as in power converters.
2. **Quasi-periodic mixed frequency/time method (MMFT):** In some circuits, the slow-scale signal path is often almost linear, while the fast-scale action is highly nonlinear. Linearity in the signal path can be exploited by expressing the slow scale components in a short Fourier series, and solving the mixed frequency/time system of equations. This Multivariate Mixed Frequency Time (MMFT) method is often more efficient for switched-capacitor filters and switching mixers.
3. **Time domain envelope methods (TD-ENV):** Envelope-type solutions can be generated from the MPDE by applying mixed initial/periodic boundary conditions. Novel time-domain methods based on FDTD or shooting along the fast time scale, and transient integration along the slow time scale, have been devised. These techniques are capable of handling circuits with nonlinearities on a fast time scale, e.g., power converters, switched-capacitor filters, switching mixers, etc..

The above numerical techniques generate sparse matrices with near diagonal or block-diagonal structure, which makes it convenient to use iterative linear solution methods (e.g., [Saa96, MFR95]) to solve large circuits efficiently.

4 MPDE-based compact modelling of LTV systems

Another useful application of the MPDE is for reduced-order modelling of linear time varying (LTV) systems. Existing reduced-order modelling techniques for linear time invariant (LTI) systems [PR90, FF95] have proven to be powerful tools for analyzing large systems efficiently. These methods produce a compact description of a large, complex system that captures the important aspects of its input-output behaviour accurately. A similar capability for LTV systems is very desirable, for many RF and communication subsystems can be modelled as linear time-varying – phenomena such as frequency translation and cyclostationary noise are captured by LTV representations. Compact descriptions of complex subsystems can be used to make bottom-up system-level design of large systems practical.

A difficulty in extending LTI reduction techniques to LTV systems has been the interference of the time-variations of the system and the input. In this section, it is shown how the MPDE can be used to overcome this difficulty and enable reduced-order modelling for LTV systems.

The time-varying small-signal equations for (3) around any large-signal excitation $b_s(t)$ and corresponding solution $x_s(t)$ are given by:

$$\begin{aligned} C(t)\dot{x}(t)+G(t)x(t)&=ru(t)\\ y(t)&=d^Tx(t)\end{aligned} \tag{5}$$

For (5), the perturbation about the large excitation is assumed to be in the form $ru(t)$, where $u(t)$ is the scalar input. A scalar output $y(t)=d^Tx(t)$ is also defined. $C(t)$ and $G(t)$ are the derivative matrices of $q(x_s(t))$ and $f(x_s(t))$. If the above equation is Laplace-transformed (analogous to the procedure for LTI systems), the system time variation in $C(t)$ and $G(t)$ interferes with the I/O time variation through a convolution. The LTV transfer function $H(t,s)$ is therefore hard to obtain; this is the difficulty alluded to earlier. The problem can be avoided by casting (5) as an MPDE:

$$\begin{aligned} C(t_1)\left[\frac{\partial\hat{x}}{\partial t_1}(t_1,t_2)+\frac{\partial\hat{x}}{\partial t_2}(t_1,t_2)\right]+G(t_1)\hat{x}(t_1,t_2)&=ru(t_2)\\ \hat{y}(t_1,t_2)=d^T\hat{x}(t_1,t_2),\qquad y(t)&=\hat{y}(t,t)\end{aligned} \tag{6}$$

(6) can also be derived from (4) by setting $\hat{b}(t_1,t_2)=b_s(t_1)+ru(t_2)$. Notice that the input and system time variables are now separated. By taking Laplace transforms in t_2 and eliminating $\hat{x}$, the time-varying transfer function $H(t_1,s)$ is obtained:

$$Y(t_1,s)=\underbrace{\left\{d^T\left[C(t_1)\left\{\frac{\partial}{\partial t_1}+s\right\}+G(t_1)\right]^{-1}[r]\right\}}_{H(t_1,s)}U(s) \tag{7}$$

$H(t_1,s)$ can be rewritten in the form:

$$H(t_1,s)=d^T\{I+s\mathcal{A}[\cdot]\}^{-1}[\tilde{r}(t_1)] \tag{8}$$

$$\text{where } \mathcal{A}[z(t_1)]=\left[C(t_1)\frac{\partial}{\partial t_1}+G(t_1)\right]^{-1}[C(t_1)z(t_1)] \tag{9}$$

$$\text{and } \tilde{r}(t_1)=\left[C(t_1)\frac{\partial}{\partial t_1}+G(t_1)\right]^{-1}[r] \tag{10}$$

(8) can be used directly by operator versions of reduced-order modelling techniques to obtain a *time-varying reduced-order model* for $H(t_1,s)$ in the form:

$$H(t_1,s)\approx\frac{\sum_{i=0}^{q-1}a_i(t_1)s^i}{\sum_{j=0}^{q}b_j(t_1)s^j}=\sum_{i=0}^{q}\frac{c_i(t_1)}{s+p_i(t_1)} \tag{11}$$

where q is a small number, much smaller than the dimension n of the original system in (6).

As an illustration, consider an explicit moment matching procedure. (8) can be expanded into a Taylor series in s with time-varying coefficients, i.e., the *moments* $m_i(t_1)$:

$$H(t_1,s) = \underbrace{d^T\tilde{r}(t_1)}_{m_0(t_1)} - s\,\underbrace{d^T\mathcal{A}[\tilde{r}(\cdot)]}_{m_1(t_1)} + s^2\,\underbrace{d^T\mathcal{A}[\mathcal{A}[\tilde{r}(\cdot)]]}_{m_2(t_1)} - \cdots \qquad (12)$$

Calculating the moments amounts to a few applications of the operator $\mathcal{A}[\cdot]$, which corresponds to time-varying linear solution of (9). This is an efficient operation that already forms the inner loop of steady-state methods like shooting or harmonic balance. Once the moments $m_i(t_1)$ are available, t_1 is set to a fixed value and any LTI model reduction technique that uses explicit moments (e.g., AWE [PR90]) can be used to obtain the coefficients in (11) at t_1. This is repeated over a range of values of t_1 to obtain the complete reduced model.

Hence explicit moment-matching for LTV reduction is achieved easily by combining existing LTI reduction techniques with efficient linear solutions of the circuit in steady state. Other reduction techniques, based on Krylov subspace methods, can also be applied to (8); these procedures have better numerical conditioning than explicit moment matching.

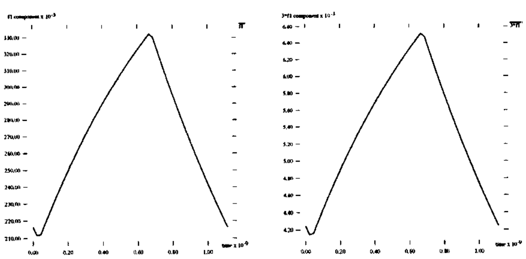
5 Application to wireless system components

In this section, the MPDE-based numerical methods described above are applied to several multi-rate circuits arising in wireless and portable applications.

5.1 Mixer simulation using MMFT

A double-balanced switching mixer and filter circuit was simulated for intermodulation distortion using the MMFT method. The RF input to the mixer was a 100kHz sinusoid with amplitude 100mV; this sent it into a mildly nonlinear regime. The LO input was a square wave of large amplitude (1V), which switched the mixer on and off at a fast rate (900Mhz).

Three harmonics were taken in the RF tone $f_1 = 100$kHz (corresponding to the t_1 variable). The LO tone at $f_2 = 900$MHz was handled by shooting in the t_1 variable. The output of the algorithm is a set of time-varying harmonics that are periodic with period $T_2 = \frac{1}{f_2}$. The first harmonic is shown in Figure 3(a). This plot contains information about all mix components of the form $f_1 + if_2$, i.e., the frequencies 900.1 Mhz, 1800.1 Mhz, etc.. The main mix component of interest, 900.1 Mhz, is found by taking the fundamental component of the waveform in Figure 3(a). This has an amplitude of 60mV.



(a) First harmonic (b) Third harmonic

Figure 3: Switching Mixer: MMFT output

The third harmonic is shown in Figure 3(b). It contains information about the mixes $3f_1 + if_2$, i.e., the frequencies 900.3 Mhz, 1800.3 Mhz, etc.. The amplitude of the 900.3 Mhz component can be seen to be about 1.1mV; hence the distortion introduced by the mixer is about 35dB below the desired signal.

The circuit was also simulated by univariate shooting for comparison. The output from shooting is shown in Figure 4. This run, using 50 steps per fast period, took almost 300 times as long as the new algorithm.

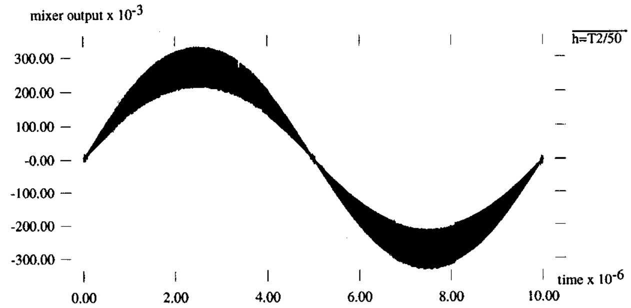
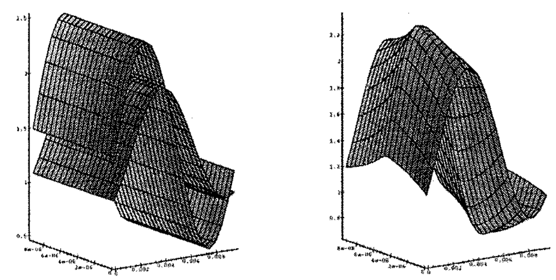


Figure 4: Mixer output from univariate shooting

5.2 S-C filter simulation using MFDTD

A switched-capacitor filter circuit was simulated using the MFDTD method. The input to the circuit was a sinusoid with 10ms period (100Hz), and the switching period was $10^{-5}s$ (100kHz). The slow time scale was taken to be t_1, and the fast time-scale was t_2. Bi-variate forms of the input and output voltages are both shown in Figure 5(a) (the larger waveform is the input). The variation along the slow time scale is apparent; note the absence of fast-time-scale variation at the output, indicating that the switching frequency has been eliminated by filtering (the input, by definition, has no fast variation). Note also the phase shift of the output with respect to the input. In contrast to the filtered output, the switches exhibit strong fast scale variation, since fast switching is key to the proper operation of the circuit. The bi-variate form of the voltage at the one of the switch capacitors is shown in Figure 5(b). Note the variation along both slow and fast time-scales. The slow time-scale variation is similar to the input, as expected, but the fast time-scale variation displays charging/discharging behaviour associated with internal losses in the switch, due to the relatively fast switching speed.

The second harmonic distortion, obtained by Fourier analysis of the output voltage along the t_1 direction, was 28dB below the fundmental.



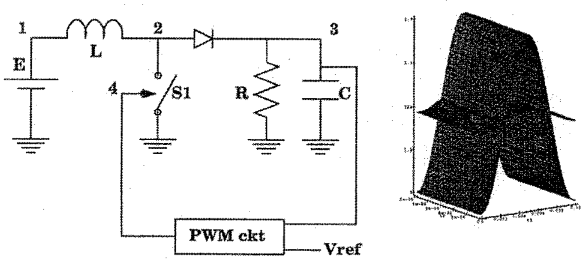
(a) input and output voltages (b) voltage at switching capacitor

Figure 5: Switched-capacitor filter: bi-variate solutions

5.3 PWM power converter simulation with TD-ENV

A boost-type DC-DC converter with PWM feedback for output voltage stabilization was simulated by TD-ENV. A simplified diagram of the circuit is shown in Figure 6(a). When the switch closes, the inductor current rises linearly until the switch opens, after which the current is diverted through the diode into the load resistor. The peak current of the inductor is related to the amount of time the switch is closed, i.e., the duty cycle of the switch control. This current determines the output voltage at node 3.

The negative feedback loop operates by comparing the output voltage at node 3 with a reference to obtain an error voltage, which is used to control the duty cycle of the control to the switch. If the output voltage is lower than the reference, the duty cycle is increased, and vice-versa. For the simulation, the input power source E was centered at 1.4V, but with a ripple of 0.8V at 100Hz added. The reference voltage for the output was also set at 1.4V. The switching rate was 100kHz. The bivariate forms of the input and output voltages are shown in Figure 6(b); the large changes are for the input waveform, and the small ones for the regulated output. Note the relative absence of fast scale variation (i.e., along the t_2 axis) of the output, indicating low ripple. The current through the inductor is shown in Figure 7. This waveform provides a useful visualisation of the operation of the converter. Note the linear charging of the inductor and the somewhat nonlinear discharge. Note also that the converter is operating in continous mode, for the current does not ever reach zero despite the fluctuations of the source battery. When the load is increased (not pictured), the inductor discharges completely for part of the fast time scale.



(a) circuit (b) input and output voltages

Figure 6: PWM DC-DC converter

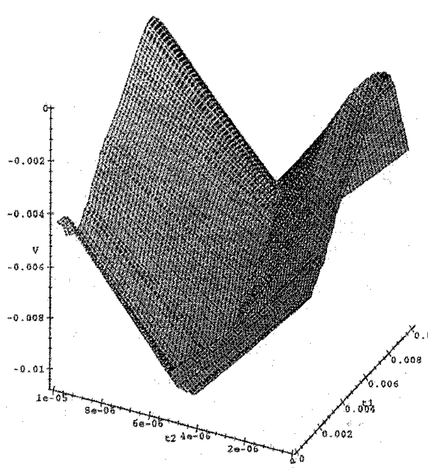


Figure 7: PWM DC-DC converter: inductor current

The dynamics of the feedback mechanism are also evident in the shape of the control voltage to the switch, shown in Figure 8. This voltage is a fast pulse train with varying duty cycle. The pulse nature of the signal is evident from the variation in the t_2 direction. The change of the duty cycle along the slow time scale can also be readily seen. This is due to feedback, which modifies the pulse width to keep the output voltage stable despite large input fluctuations.

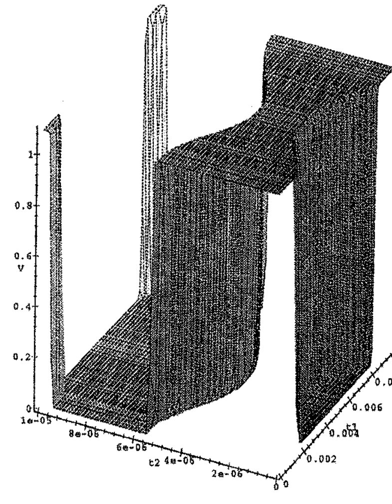


Figure 8: PWM DC-DC converter: switch control

6 Conclusion

We have presented efficient MPDE-based numerical simulations of switching mixers, switched-capacitor circuits and switching power converters, circuits that are difficult to simulate by traditional methods. The new methods are much faster than previous ones (e.g., 300x speedup) and also more accurate. Further, they can provide results in three-dimensional form, which is a powerful new way of visualizing the operation of circuits with widely separated time scales. We have also shown how the MPDE can be used for LTV model reduction. This capability enables compact system-level abstraction of RF circuit blocks, with significant application in hierarchical design methodologies for wireless systems.

Acknowledgments

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