An Efficient and Robust Technique for Tracking Amplitude and Frequency Envelopes in Oscillators

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Abstract— **Envelope-following methods face special challenges when applied to oscillators because of their fundamental property of dynamically-changing frequencies. In this paper, we present a novel and robust approach for oscillator envelope following. Our method combines, unifies and extends ideas from two prior oscillator envelope-following approaches, Petzold's method and the WaMPDE. Our technique uses two extra system unknowns, as well as two extra "phase condition" equations, to track quantities related to dynamical frequency/time-period changes. These advances confer significant robustness without appreciable computational overhead. We validate our method on LC, ring and crystal oscillators, predicting frequency and amplitude modulations as well as transient startup envelopes accurately. Speedups of 1-2 orders of magnitude are obtained over traditional alternatives.**

I. Introduction

Oscillators are important in many engineering and communication systems. For example, they are often used as time references in digital circuits or for information encoding in communication systems. As is well known, oscillator simulation presents challenges that traditional SPICE-like simulation (*e.g.*, [1], [2]) is incapable of addressing effectively. Due to their marginal stability [3], small phase errors accumulate unboundedly during transient simulation. This leads to a much worse tradeoff between simulation timestep size and accuracy for oscillators than for non-autonomous circuits. This is especially true for high-Q oscillators, which often feature very slow and sensitive amplitude responses. To obtain even reasonably accurate results, extremely small step sizes can be required during simulation.

The *envelope* of a highly oscillatory signal refers to its slowly-varying characteristics, such as the gradual amplitude or frequency modulation of fast oscillations. When very slow envelopes are present in an oscillator, predicting waveforms by conventional time-stepping simulation can be extremely inefficient because of the widely separated time scales of the fast and slow components [4]. Such problems are encountered in many practical design situations: *e.g.*, when simulating startup/shutdown of oscillators (especially high-Q ones), frequency modulation in voltage-controlled oscillators (VCOs), phase-locked loops (PLLs), injection pulling/locking by external signals, *etc.*. It should be noted that in such situations, designers are often directly interested in the slow envelopes themselves.

A variety of methods have been devised to solve for envelopes more efficiently than transient simulation. The majority of such techniques have focussed on circuits that are not oscillators, *i.e.*, where the circuit's frequency does not change. The earliest such technique, to our knowledge, is the time-domain envelope-following method proposed by Petzold in [5], which was later adapted for circuit simulation (*e.g.*, [6]–[10]) with application to both transient and steady state simulations. Another class of techniques, Fourier-envelope methods [11]–[13], combines frequency-domain Harmonic Balance (HB) and time-domain integration methods. These techniques solve for the slowly-varying Fourier coefficients of fast oscillations. Recently, a family of methods based on Multi-time Partial Differential Equations (MPDEs) [4], [14], [15] has also emerged. These methods rely on separating slow and fast variations by employing several artificial time variables.

For oscillators, envelope-following methods present special challenges, as may be expected because their time-periods (or frequencies) change dynamically during operation. The changing frequency or time period must be calculated very accurately in order to track envelopes effectively; doing so correctly, in conjunction with amplitude envelopes, constitutes the main challenge for oscillator envelope simulation. In [5], Petzold proposed a technique for tracking the changing period by minimizing the difference between values at the beginning and the end of a period. Since the cost of such minimization (via optimization) can be high, Gear [16] proposed a heuristic to alleviate this issue, identifying periods using zero-crossings. A remaining issue in both approaches concerns the correct choice of envelope step sizes when the oscillator's period varies over time. In our experience, correct choice of envelope step sizes, to correspond to an integral number of fast cycles, is critical for robustness in oscillator envelope following.

Another approach towards oscillator envelope is based on generalizing MPDE approaches using the notion of "warping" time to even out the changing periods of fast cycles. This approach, termed the WaMPDE [17], is appealing not only because of its theoretical elegance, but its use of an explicit unknown variable, the *local frequency*, that is solved for along with all other waveforms to capture the slowly changing time-period (or frequency) of the oscillator. An issue with the WaMPDE, however, is that of choosing envelope initial conditions [18]; unsuitable choices can severely compromise the computational efficiency that is the primary motivation for envelope simulation in the first place.

In this paper, we present a new oscillator envelope method which combines the advantages of Petzold-style methods and the WaMPDE, while eliminating their disadvantages. From the Petzold point of view, our technique automatically chooses correct envelope steps, regardless of dynamic frequency changes; from the WaMPDE point of view, the envelope initial condition problem is sidestepped, while the notion of using system unknowns to capture changing frequencies is retained and generalized. The key advance in our method is to add *two* extra system unknowns, one representing the changing frequency/period of the oscillator (as in the WaMPDE), the other to estimate the envelope timestep. To obtain a "square system" and be able to solve uniquely for these unknowns, we add two "phase condition" equations. We term our method "Multiple Phase Condition based ENVelope following" or MPCENV for short.

Because it does not rely on minimization, MPCENV's computational efficiency is similar to that of non-oscillatory envelope methods. By automatically taking good care of the problem of choosing envelope steps correctly via the extra variables, MPCENV achieves unprecedented robustness in oscillator envelope following, while retaining the computational efficiency expected of envelope methods. We validate MPCENV on several types of LC- and ring oscillators, analyzing frequency and amplitude modulation as well as slow startup transients. Our simulations confirm excellent matches between MPCENV and carefully conducted traditional transient runs. We obtain speedups of 1–2 orders of magnitude.

The remainder of the paper is organized as follows. In Section II, we briefly review envelope-following methods and discuss issues for

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oscillator envelopes. In Section III, we provide related background on the MPDE and the WaMPDE methods. In Section IV, we introduce the MPCENV method and discuss its relation to the WaMPDE. In Section V, we present results from applying MPCENV to several oscillators and VCOs, to investigate startup transients, amplitude and frequency modulation, *etc.*.

II. Envelope-following Methods

In this section, we first review existing envelope-following methods for non-oscillatory circuits, and then discuss extensions to oscillators [5], [16].

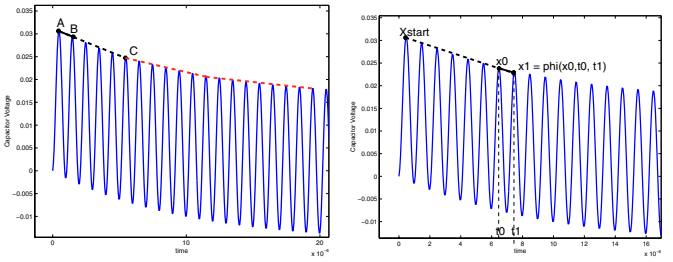
A. Non-autonomous envelope following methods

A circuit can be described by the system of differential equations

$$\dot{q}(x) + f(x) = b(t), \tag{1}$$

where x is a vector of state variables (such as node voltages and branch currents), q contains capacitor charge or inductor flux terms, and f represents resistive terms [15].

We assume that the solution of the circuit to be simulated has fast oscillations whose amplitude changes much more slowly than the oscillations themselves. When this solution is sampled at every fast oscillation period T, the resulting samples can be interpolated using a slowly varying curve, termed the *envelope*. The basic idea of envelope-following methods is shown in Figure 1(a), using a scalar DAE for illustration. We start our simulation at time t_0, at which the state variable has value x_0 (point A in the figure). Transient simulation is performed accurately for one cycle of the fast oscillation and the state variable is now at point B shown in the figure. A secant line between A and B is drawn and then extrapolated over a large "envelope timestep", which can comprise many fast cycles, to reach the solution at point C. This process is repeated until the end of the simulation interval.



(a) Forward-Euler based envelope-following method.

(b) Backward-Euler based envelope-following method.

Fig. 1. Illustration of envelope and Petzold's method.

The process described above is analogous to solving the envelope by conventional forward-Euler integration. It uses the slope between point A and B to approximate the derivative of the envelope solution at point A. This is a valid approximation if the envelope varies much more slowly than the fast oscillation. If the extrapolation distance is $(m-1)T$ (time interval between point B and C), the process of envelope-following can be described by a difference equation:

$$x(t+mT) = x(t) + mT\frac{x(t+T)-x(t)}{T}. \tag{2}$$

Although this forward-Euler based envelope-following is easy to illustrate, it is not very useful in practice since the envelope step cannot be very large due to stability issues, just as with the normal forward-Euler integration method. A more stable, backward-Euler based, envelope-following method is given by

$$x(t+mT) = x(t) + mT\frac{x(t+mT)-x(t+(m-1)T)}{T}. \tag{3}$$

When implicit methods are used, the only unknown is $x(t+(m-1)T)$; the state at $t+mT$ can be evaluated by integrating (1) for one fast cycle, and can be written using the state transition function ϕ as

$$x(t+mT) = \phi(x(t+(m-1)T), t+(m-1)T, t+mT). \tag{4}$$

(3) is a boundary-value problem, illustrated in Figure 1(b), which can be solved by any nonlinear solution method, such as Newton-Raphson. An initial value of unknown x_0 is guessed and then a cycle of transient simulation is performed to obtain the state x_1. This information is used to update x_0 using Newton's method until the boundary condition $x_1 - x_{start} = m(x_1 - x_0)$ is satisfied.

B. Extensions to oscillators

Envelope-following methods involve integrating (1) for one cycle T. For oscillators, however, T is not known a-priori. It is extremely important that T be accurately calculated; otherwise, inaccuracies in envelope following may build up rapidly to such an extent that large envelope steps cannot be taken at all, leading to a complete breakdown of any envelope algorithm.

In [5], Petzold proposed a minimization-based algorithm to estimate the period. It is motivated by the observation that if $x(t)$ is periodic with period T, then $||y(t+T)-y(t)|| = 0$ on the interval on which $x(t)$ is defined. Therefore, the period T can be estimated by minimizing $||y(t+T)-y(t)||_2$. However, performing such a minimization is not computationally cheap, and also, reliant on heuristics for effective minimization.

Another, simpler, approach towards defining the period is to find certain points that appear repeatedly in the waveform; for example, zero-crossing points [16]. In [16], zero-crossing of the derivatives is used, since the waveform itself may not actually cross zero. At the same time, $||y(t_1)-y(t_2)||$, where t_1 and t_2 are zero-crossings, is also examined in case the waveform has more than one zero-crossing point within a period.

However, estimation of the envelope step is left to the time-stepping algorithm, just as in normal numerical solution of differential equations, in both the methods above. This can be a cause of reduced robustness when the time-period is varying, because the envelope step may not exactly be an integer number of cycles. It is critical that this envelope step be accurately estimated, for essentially the same reason that T needs to be found accurately, *i.e.*, in order to properly "line up" the points A, B, and C in Figure 1(a) and obtain a smooth envelope.

III. MPDE-based Envelope Methods

In this section, we first review MPDE methods and their numerical solution, including the problem of finding good initial conditions. We then present their extensions to oscillators, *i.e.*, the Warped MPDE (WaMPDE).

A. MPDE methods for widely separated time scales

In the multi-time partial differential equation (MPDE) formulation, artificial time scales are introduced to decouple slow and fast time scales [4], [15]. Each rate of variation is represented by its "own" time scale, so can be solved using its "own" time step size. Thus, the envelope, *i.e.*, the slow component in a signal, can be solved efficiently since large time steps can be used for its time scale. For simplicity, we consider two time scales, one fast and one slow. The MPDE form corresponding to (1) is

$$\frac{\partial q(\hat{x})}{\partial t_1} + \frac{\partial q(\hat{x})}{\partial t_2} + f(\hat{x}) = \hat{b}(t_1, t_2), \tag{5}$$

where $\hat{x}(t_1,t_2)$ and $\hat{b}(t_1,t_2)$ are the bivariate forms of $x(t)$ and $b(t)$, respectively. Here, without loss of generality, we choose t_1 to be the slow time scale and t_2 to be the fast time scale.

The envelope equation that results from the MPDE [18] is essentially a DAE in the slow envelope time scale. Taking a time-step along the envelope time scale involves solving a steady-state problem with periodic boundary conditions along the fast time scale [15]. To start the DAE solution along the envelope time scale, however, a periodic solution along the fast time scale needs to be given at $t = 0$ (the initial starting point of the envelope simulation) along the envelope time scale. It has been shown [18] that proper choice of this envelope initial condition, which involves heuristics, is crucial to the efficiency of the MPDE-based and Fourier envelope solution.

B. The Warped MPDE (WaMPDE)

The MPDE is not well suited for analyzing frequency modulation (FM) in oscillators. To remedy this situation, the Warped MPDE (or WaMPDE) formulation was devised in [17]. In the WaMPDE, the fast time scale is dynamically rescaled (or warped) to undo frequency-modulation and make the fast undulations uniform. The resulting warped multi-variate waveforms can be represented compactly (as for circuits without FM), while the rescaled fast time scale captures the effects of FM. At the equation level, the WaMPDE is formed by adding an extra unknown (representing the instantaneous frequency) to the MPDE as [17]:

$$\frac{\partial q(\hat{x})}{\partial t_1} + \omega(t_1)\frac{\partial q(\hat{x})}{\partial t_2} + f(\hat{x}) = \hat{b}(t_1,t_2), \tag{6}$$

where t_1 is the unwarped (slow) time scale and t_2 is the warped (fast) time scale.

To solve the WaMPDE, an extra equation must be added to the system, since there is one more unknown than there are equations. We note that (6) is autonomous in the t_2 scale, *i.e.*, a time shift in t_2 from any solution $x(t_1,t_2)$ is also a valid solution. Uniqueness of the solution is enforced by adding a phase condition, which fixes the phase of one variable at, *e.g.*, $t_2 = 0$. For example, the phase condition can be specified to be:

$$\frac{d\hat{x}_l(t_1,t_2)}{dt_2}|_{t_2=0} = 0, \tag{7}$$

where $\hat{x}_l(t_1,t_2)$ is one of the state variables. The WaMPDE can then be solved with numerical methods similar to those for the MPDE.

The WaMPDE also faces the same problem of finding good envelope initial conditions, just as for the MPDE. Finding good envelope ICs for oscillators is, indeed, typically more difficult, due to the unknown period of the oscillator. The techniques in [18] are not immediately extensible to oscillators.

IV. The Proposed Algorithm

In this section, we propose a *M*ultiple *P*hase *C*ondition based *ENV*elope-following method (MPCENV) for oscillators. The connection between MPCENV and the WaMPDE is also discussed.

A. The MPCENV method

In MPCENV, we introduce two extra unknowns: T to represent the changing period of the oscillator, and T_{ext} (the envelope step) to capture the effect of FM — since the periods of small cycles will typically vary, although slowly, within an envelope step. In other words, the envelope step will, in general, no longer remain an integer number of the period T (at any given time point). With these two extra unknowns, the envelope-following equation system (for example, the backward-Euler based one (3)) becomes underdetermined, since the number of unknowns becomes more than that of equations.

Our solution to this is directly motivated by our goal of keeping the phases at the beginning and the end of a fast cycle (also the beginning of the next cycle) the same. Recall that for the non-oscillatory case, this is automatically satisfied because the period of the fast oscillation is known (and a fixed constant over the simulation). For the oscillator, since the period is unknown and changing, these conditions must be enforced. For example, we can use a phase condition similar to (7), at both the beginning and the end of the fast cycle over which standard transient simulation is performed during envelope following. Only the phase of one of the state variables needs to be fixed. Using the backward-Euler based envelope-following method as an example, we add these two phase conditions to x_0 and x_1, as shown in Figure 1(b).

We now have a well-determined system with equal numbers of equations and unknowns, which can be solved by nonlinear solvers such as Newton's method. The augmented backward-Euler based envelope-following equation system is:

$$\begin{aligned} \frac{x(t+T_{ext}+T)-x(t+T_{ext})}{T} &= \frac{x(t+T_{ext})-x(t)}{T_{ext}} \\ \frac{dx_l}{dt}|_{t+T_{ext}} &= 0 \\ \frac{dx_l}{dt}|_{t+T_{ext}+T} &= 0, \end{aligned} \tag{8}$$

where x_l is the state variable to which the phase constraints are applied.

Note that the initial condition $x_l(0)$ should be chosen such that it also satisfies the phase condition applied. This is not difficult to achieve; for example, a transient simulation can be run for a few initial cycles to choose an appropriate point for this initial condition. We note that this idea is closely related to a published method for finding MPDE initial conditions [18].

B. Numerical solution of MPCENV

MPCENV is a boundary value problem with 2 more phase constraints and can be solved by Newton's method. At each MPCENV step, the unknowns are $x(t+T_{ext})$, T and T_{ext}. $x(t+T_{ext}+T)$ can be evaluated by integrating one cycle T starting from $x(t+T_{ext})$. We denote the unknown state variables $x(t+T_{ext})$ by x_0; the corresponding time point as t_0 ($t_0 = t+T_{ext}$); the starting state $x(t)$ (which is either an IC, given at $t = 0$, or known from the previous step) as x_s. We can rewrite (8) using state transition function notation as:

$$\begin{aligned} \frac{\phi(x_0,t_0,t_0+T)-x_0}{T} &= \frac{x_0-x_s}{T_{ext}} \\ \frac{dx_{0l}}{dt} &= 0 \\ \frac{d\phi(x_0,t_0,t_0+T)_l}{dt} &= 0. \end{aligned} \tag{9}$$

The calculation of the Jacobian matrix involves the evaluation of the derivative of the state transition function, also known as the sensitivity matrix. It represents the sensitivity of $\phi(x_0,t_0,t_0+T)$ to changes of both x_0 and T. The evaluation of the sensitivity matrix is performed during transient simulation with a little additional computation, just as for the shooting method [19]. Indeed, if we assume that the sampled envelope does not change with time, then the boundary condition in (8) becomes a simple periodic one, resulting in the shooting method.

We apply the trapezoidal integration method (TRAP – better suited for oscillators since overly stable methods like backward-Euler can damp out oscillations) to integrate (1) from t_0 to t_0+T. Both $\frac{d\phi}{dx_0}$ and $\frac{d\phi}{dT}$ can be derived as described in [20]. If x_n is the state at t_n ($t_0 \le t_{n-1} < t_n \le t_0+T$), then

$$\frac{dx_n}{dx_0} = (\frac{C_n}{h_n}+\frac{G_n}{2})^{-1}(\frac{C_{n-1}}{h_n}-\frac{G_{n-1}}{2})\frac{dx_{n-1}}{dx_0}, \tag{10}$$

where $C_i = \frac{dq(x_i)}{dx_i}$, $G_i = \frac{df(x_i)}{dx_i}$ and $h_i = t_i - t_{i-1}$. Note that $\frac{C_n}{h_n} + \frac{G_n}{2}$ is the Jacobian matrix of (1) during the integration and is already available from transient simulation. Starting from $\frac{dx_0}{dx_0} = I$, $\frac{d\phi}{dx_0}$ can be found by applying (10) repeatedly until $t_0 + T$ is reached.

Similarly, $\frac{d\phi}{dT}$ can be found by applying

$$\frac{dx_n}{dT} = (\frac{C_n}{h_n} + \frac{G_n}{2})^{-1} \\ [(\frac{C_{n-1}}{h_n} - \frac{G_{n-1}}{2})\frac{dx_{n-1}}{dT} + \frac{q(x_n) - q(x_{n-1})}{T}] \quad (11)$$

repeatedly, starting from $\frac{dx_0}{dT} = 0$.

V. Applications and Validation

In this section, we apply and validate MPCENV on LC and ring VCOs. We investigate frequency modulation as well as amplitude envelopes, and also simulate startup transient envelopes in a high-Q, crystal-based oscillator. MPCENV results show excellent matches with those from traditional SPICE-like transient simulation, while delivering speedups of 1 -2 orders of magnitude. (All simulation were performed using MATLAB on an 2.4GHz PC running Linux.)

A. LC VCO

A simple LC VCO from [17] is shown in Figure 2 and simulated using MPCENV. The oscillator contains an LC tank with the capacitance controlled by a voltage source. The element values are: $R = 1k\Omega, Cd = 0.3uF, C = \frac{1}{2\pi}10^{-7}F, L = \frac{1}{2\pi}10^{-7}H, Cm = \frac{1}{4\pi}10^{-7}F$. The nonlinear negative resistor characteristic is given in [17] as

$$i = f(v) = (G_0 - G_\infty)V_k \tanh(\frac{v}{V_k}) + G_\infty v, \quad (12)$$

where $G_0 = -0.1, G_\infty = 0.25$, and $V_k = 1$. The oscillator has a nominal frequency of 10MHz.

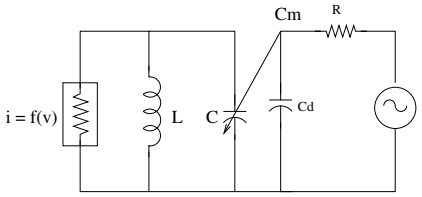
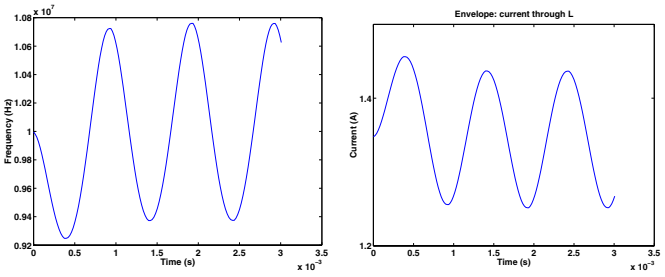


Fig. 2. Circuit schematic of a LC VCO.

We start the envelope simulation from the oscillator's steady state. Initial conditions are chosen so that they satisfy the phase condition in (8). The controlling voltage is a sinusoid with the frequency 10^4 times slower than the oscillator's free-running frequency ($V_c = 0.6sin(2\pi 10^3 t)$). The main purpose of this simulation is to illustrate strong FM in VCOs and show how MPCENV captures it. Figure 3(a) shows the frequency change due to the variation of the capacitance in LC tank. The envelope solution of the inductor current from MPCENV is shown in Figure 3(b). The full transient simulation result is compared with the result of MPCENV — the envelopes match perfectly. The full simulation result is not depicted here due to the density of fast oscillations. In this example, MPCENV takes envelope steps of about fast 200 cycles each, solving the Newton equations with only 2-3 iterations at each step. A speedup of more than 40× over full transient simulation is obtained for this example.

B. 3 stage ring VCO

A 3 stage ring VCO is shown in Figure 4. Each stage is identical in this VCO. The resistance is varied by changing the controlling voltage; this changes the period/frequency of the oscillator. The oscillator has a free-running frequency of 100MHz.



(a) MPCENV solution (b) envelope solution

Fig. 3. LC VCO: solution of the inductor current.

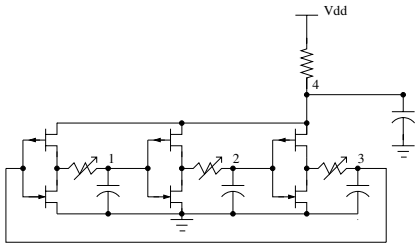
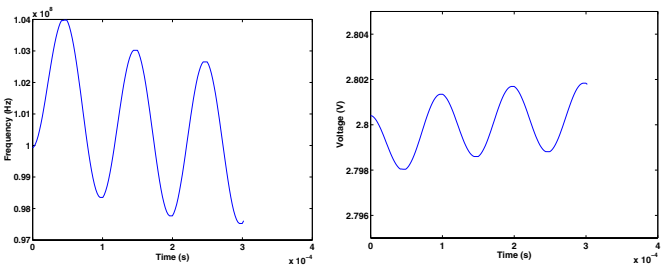


Fig. 4. Circuit schematic of a ring VCO.

We choose a point in the oscillator's steady state as the initial condition for MPCENV, *i.e.*, we start the simulation from steady state. The VCO's controlling voltage is $10 + 2sin(2\pi 10^4 t)$. The simulation interval is 0.3 ms. A nominal envelope timestep of 200 cycles is used in this example. Figure 5(a) shows the variation of the oscillation frequency, as it responds to the controlling voltage. The envelope solution of the amplitude variation at the output of the 3rd stage is shown in Figure 5(b) and compared against transient simulation results. They are in good agreement. We obtain speedups of about 35× for this example.



(a) Ring VCO: frequency modulation. (b) MPCENV envelope solution of the output of the 3rd stage.

Fig. 5. Ring VCO: solutions of MPCENV.

C. Startup transient, Pierce crystal oscillator

Figure 6 shows a Pierce crystal oscillator from [21], [22]. The element values are: $R_1 = 100K\Omega, R_2 = 2.2K\Omega, C_1 = 100pF, C_2 = 100pF, C_p = 25pF, C_s = 99.5fF, R_s = 6.4\Omega$ and $L_s = 2.55mH$, resulting in a high quality factor Q about 2.5×10^4. The bipolar transistor has current gain $\beta = 100$, and the oscillator's nominal frequency is around 10MHz. Due to its high Q, the crystal oscillator takes many oscillatory cycles to reach its steady state from power-on start up.

In the simulation, we use variable envelope stepsizes, based on a very simple convergence criterion: if the envelope Newton converges in a few iterations, we increase the envelope step; otherwise, if Newton takes too many iterations, we shorten the step. Figures 7–8

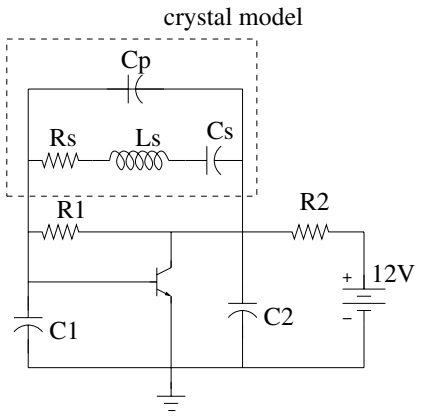
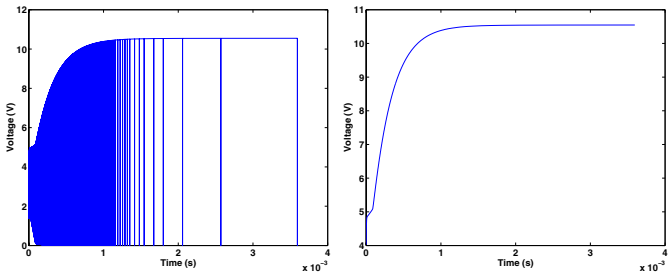


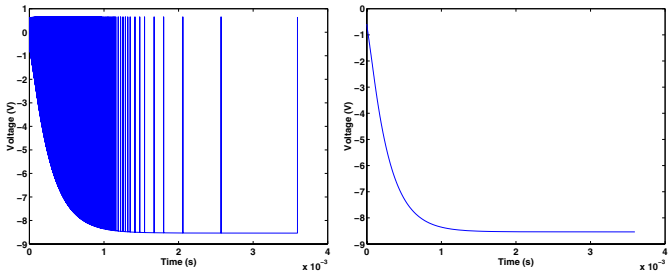
Fig. 6. Circuit schematic of a pierce crystal oscillator.

show waveforms obtained by MPCENV at the base and the collector of the BJT. As can be seen, the envelope step is small at the beginning due to a relatively fast-changing envelope. The envelope step gets larger as the oscillator approaches its steady state and the waveform stabilizes. Over the simulation, MPCENV takes an average envelope step of about 91 fast cycles. For full transient simulation, it takes about 1ms (10000 cycles) to approach the steady state and about another 2ms (20000 cycles) to actually reach the steady state. We obtained a speedups of 45 over the transient for this simulation.



(a) MPCENV solutions (b) the envelope solution

Fig. 7. Waveform at the collector of the BJT in the pierce crystal oscillator.



(a) MPCENV solutions (b) envelope solution

Fig. 8. Waveform at the base of the BJT in the pierce crystal oscillator.

We note in passing that MPCENV can thus also be used as a means for finding the steady state solution of high-Q oscillators (*e.g.*, [10]), since it accelerates the simulation of startup transients — although this is of course not its only capability.

VI. CONCLUSIONS

We have presented a robust and efficient algorithm for oscillator envelope following. Our approach is able to capture dynamic frequency changes accurately and robustly, leading to unprecedented robustness compared to prior methods. We have applied our method to different types of oscillators and obtained speedups of 1-2 orders of magnitude over transient simulation.

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