Obtaining Frequency Sensitivities to Variations Analytically from Parameterized Nonlinear Oscillator Phase Macromodels

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***Abstract*— System-level variability analysis and design centering for oscillators relies on fast and accurate methods for obtaining the parametric sensitivities of higher-level performances (such as center frequency) directly from phase macromodels. We present an efficient and elegant method, involving no numerical simulation, for finding parametric sensitivities of oscillator frequencies directly from nonlinear phase domain macromodels. We validate the method, termed FS-PPV, on numerically extracted ring and LC oscillator PPV macromodels, as well as on a purely analytical exact PPV macromodel for idealized ring oscillators. We apply FS-PPV to find statistical distributions of oscillator center frequencies and validate these distributions against Monte-Carlo simulations. FS-PPV achieves speedups of more than 3000× over brute-force Monte-Carlo based parameter variation analysis.**

I. INTRODUCTION

Oscillators are fundamental objects in many physical and engineering domains – for example, in electronics (*e.g.*, VCOs, PLLs), mechanics (*e.g.*, mechanical clocks), optics (*e.g.*, lasers) and biology (*e.g.*, circadian rhythms, neural signaling, heart muscle cells). The free-running frequency of an oscillator is one of its most important properties. In high-speed digital, mixed-signal, RF and analog circuits alike, a central goal of oscillator design is to accurately place, and to stabilize, its free-running or "center" frequency. The exact dependence of center frequency on oscillator design parameters is typically complex – for example, in on-chip LC oscillators, it depends not only on nominal LC tank parameters but also on device/interconnect parasitics and the nonlinear gain properties of active devices. The dependence of center frequencies on semiconductor device parameters and parasitics is even more acute for ring oscillators, which are of central importance in most integrated applications. Because of the non-trivial dependence of frequency to a variety of parameters, accurate tools for finding the relationship quickly and accurately play an especially critical rôle in oscillator system design.

As is well known, technology scaling from the 90nm to the 65nm node in recent years (with 45nm and 32nm on the way) [1] has led to a dramatic increase in the importance of manufacturing tolerance and parameter variability issues. At these feature sizes, layout geometries and process parameters are subject to large variations. Underlying lithographic and process parameter variations translate to uncertainties in electrical parameters such as threshold voltage, parasitic capacitances, *etc.*, which, in turn, impact the center frequencies of oscillators. It is crucially important during design to quickly but accurately obtain information about yield and variability with regard to center frequency. In this context, random dopant effect[1] and surface roughness[2] are two effects of particular concern, since they directly impact threshold voltages of MOS devices.

Being able to predict the effects of variability properly depends on several components: accurate deterministic and/or statistical characterization of all the varying parameters, good device models, and efficient analysis/simulation algorithms for predicting circuit performance variability and yield [2], [3]. The varying parameters are ultimately a set of physical parameters (*e.g.*, doping levels and profiles, geometrical dimensions *etc.*), each one of which involves both correlated and uncorrelated variations [2]. For example, the variation of dopant density level includes both a correlated part — which comes from global variations (*e.g.*, die-to-die doping variations) or deterministic variations (*e.g.*, dopant gradients across chip) — and an uncorrelated part, which comes from local random variability (*e.g.*, random doping fluctuation) and systematic effects that are inadequately modelled (*e.g.*, dopant variations that are sensitive to proximity effects) [2], [4]. Physical parameters with such variations affect important electrical parameters, such as MOSFET threshold voltage. Good device models, featuring physically based parameters, are important for proper characterization of variability – for example, models such as EKV and PSP are preferred over BSIM, while features hundreds of empirically fitted, redundant parameters.

[1]Typical numbers of dopant atoms in a MOS channel now number under 100 atoms.

[2]Gate oxide thicknesses today correspond to about 10 molecules of SiO_2.

The most commonly used analysis method for obtaining circuit performance statistics is the Monte Carlo method. For analyzing frequency variability, Monte-Carlo consists simply of randomly sampling parameters according to specified probability distributions, finding the center frequency of the oscillator for the given parameter choices (using a simulation algorithm such as harmonic balance [5]) and thus obtaining a distribution of center frequencies. The technique is extremely general and broadly applicable; however, it suffers from being very computationally expensive since it typically requires very large numbers of samples[3]. A widely used alternative approach is to first approximate the relationship between parameters and circuit performances (*e.g.*, an oscillator's center frequency) as a simple linear function, specified by the *sensitivities* of the performance to each parameter of interest. This linear function, once available, can be evaluated rapidly for any given choice of parameters; more importantly, analytical or fast computational methods can map statistics or min/max bounds of the parameters directly to statistics/bounds of performances.

Given a SPICE-level circuit for an oscillator, finding the sensitivities of its center frequency with respect to parameters is a long-solved problem; adjoint sensitivity techniques in conjunction with oscillator steady state computation methods like harmonic balance and shooting [5], [6] are typically used. However, this capability is not sufficient for variability analysis of large on-chip systems today, which are typically comprised of many hierarchical circuit blocks. For system level simulations, it is common practice to use *macromodels* to replace these blocks in order to make computation tractable [7]–[10]. This is especially the case for oscillator-based systems, where phase-domain macromodels — particularly, nonlinear time-shifted PPV macromodels [11]–[13] — are heavily used. Just as for individual circuits, it is important to assess the impact of variability on systems comprised of macromodels. Indeed, it is common for the SPICE-level circuit of an oscillator not to be available to a system simulator (*e.g.*, due to intellectual property concerns); only the macromodels are available.

For variability analysis of systems comprised of macromodels, the macromodels must themselves be parameterized. While the problem of coming up with good parameterized macromodels is itself a difficult one in general, we have recently been successful in developing a general procedure for rapidly extracting accurate parameterized PPV macromodels for any oscillator [14]. By employing the parameterized PPV macromodel in transient or harmonic balance (HB) simulations [14], [15], it is possible to calculate the impact of variability in the parameters on center frequency changes. This process does not directly provide center frequency sensitivity information, however; computing sensitivities via finite differences on transient/HB simulations is cumbersome and can suffer from accuracy/roundoff errors.

In this paper, we present a simple and elegant means to calculate the frequency sensitivity of any oscillator from a parameterized PPV macromodel. Our method, termed FS-PPV, is based on an exact analytical result that we derive: the center-frequency sensitivity of an oscillator is simply the average over one cycle of the parameterized PPV. In addition to its computational utility, the simplicity of this result translates into increased design insight.

We demonstrate the use of FS-PPV for ring and LC oscillators, with numerically obtained PPVs as well as with purely analytical PPVs for an idealized 3-stage ring oscillator. We firstly apply our new method to a three-stage ring oscillator with ideal abruptly-switched inverter. FS-PPV is validated by comparing the derived analytical expression of frequency sensitivity to that in the previous work. Secondly, taking two practical oscillators – a cross-coupled LC oscillator and a 3-stage ring oscillator – we use FS-PPV to calculate their frequency sensitivities and derive an simple expression for the PDF of frequency assuming Gaussianly distributed, uncorrelated[4] We demonstrate that the output PDF matches results from Monte Carlo analysis.

[3]its accuracy improves only as the square root of the number of samples (*i.e.*, relatively slowly).

[4]Correlated parameters are also handled easily, using principal component analysis (PCA).

The remainder of the paper is organized as follows. In Section II, we briefly review the parameter variability aware PPV macromodel [14]. In Section III, we derive an analytical expression calculating the frequency sensitivity. In Section IV, we validate our new method by applying it analytically to an idealized ring oscillator and comparing with a previously known analytical result for this oscillator. In Section V, we validate FS-PPV on two practical oscillators, find center frequency PDFs, and compare against the Monte Carlo method.

II. Previous Work - Parameter Variability Aware, Nonlinear Time-Shifted Oscillator Macromodels

In this section, we briefly introduce the parameter-variability equiped PPV macromodel.

A general oscillator can be described by an ODE equation,

$$\dot{x}(t) + f(x(t), p^*) = 0, \tag{1}$$

in which p^* stands for the nominal parameters. It has been proved [14] that, the steady-state solution under a small parameter variation Δp can be expressed as

$$x_p(t) = x_s(t + \alpha(t)) + y(t + \alpha(t)), \tag{2}$$

where $x_s(t)$ is the steady-state solution of (1) and $y(t + \alpha(t))$ is the amplitude variations which can be omitted when parameter variation is small [14]. The time shift $\alpha(t)$ in this solution abides by

$$\dot{\alpha}(t) = -v_1^T(t + \alpha(t)) SF_p(t + \alpha(t)) \Delta p, \tag{3}$$

where $v_1^T(t)$ is the PPV [10] and $SF_p(t)$ is the sensitivity function defined by

$$SF_p(t) = \frac{\partial f}{\partial p} \mid_{x_s(t), p^*} . \tag{4}$$

This PPV macromodel enables parameter-variability aware simulation of oscillator phase, hence frequency variance *wrt* parameter variation can be obtained from the phase waveform and the frequency sensitivity is estimated by numerical differentiation. However, the numerical differentiation introduces unwanted round-off or approximation error, and the byproduct, phase waveform, is unnecessary. So we try to avoid simulating the phase waveform, and find a direct analytical expression of the frequency sensitivity.

III. PPV Based Analytical Equation for Calculating Frequency Sensitivity

In this section, we derive an nice analytical expression for the frequency sensitivity, which is only one simple integration of the parameter-variability equiped PPV macromodel.

We define that the nominal frequency f_0 shifts Δf under a small variation of the parameter Δp. The new frequency is

$$f_1 = f_0 + \Delta f. \tag{5}$$

Using the parameter-variability aware PPV macromodel discussed in Section II, the time shift $\alpha(t)$ abides by the equation

$$\dot{\alpha}(t) = -v_1^T(t + \alpha(t)) SF_p(t + \alpha(t)) \Delta p. \tag{6}$$

New frequency being $f_1 = f_0 + \Delta f$, the time shift $\alpha(t)$ can also be expressed as

$$\alpha(t) = \frac{1}{f_0}[\Delta f \cdot t + q(f_1 t)], \tag{7}$$

where $q(\cdot)$ is a 1-periodic function.

Hence, the increase of the time shift $\alpha(t)$ within a new period $T_1 = 1/f_1$ is

$$\alpha(t + \frac{1}{f_1}) - \alpha(t) = \frac{\Delta f}{f_0 f_1}. \tag{8}$$

We can also calculate this increase by integrating the $\dot{\alpha}(t)$ using (6):

$$\alpha(t + \frac{1}{f_1}) - \alpha(t) = \int_0^{\frac{1}{f_1}} -v_1^T(t + \alpha(t)) SF_p(t + \alpha(t)) \Delta p dt, \tag{9}$$

in which $v_1^T(t)$ and $Sf(t)$ are both f_0-period function. For simplicity, we define a 1-periodic function $\chi(\cdot)$ by

$$\chi(f_0 t) = -v_1^T(t) SF_p(t). \tag{10}$$

Then (9) becomes

$$\alpha(t + \frac{1}{f_1}) - \alpha(t) = \int_0^{\frac{1}{f_1}} \chi(f_0(t + \alpha(t))) \Delta p dt. \tag{11}$$

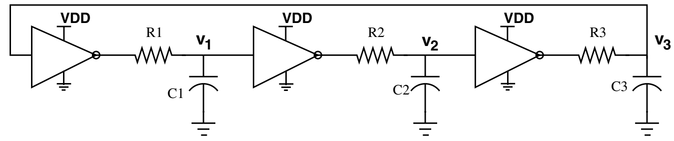

Fig. 1. Idealized ring oscillator.

Replacing $\alpha(t)$ in the right hand side using (7), and we have

$$\alpha(t + \frac{1}{f_1}) - \alpha(t) = \int_0^{\frac{1}{f_1}} \chi(f_1 t + q(f_1 t)) \Delta p dt. \tag{12}$$

When Δp is 0, $\alpha(t)$ remains zero. So, it's reasonable to say that $q(f_1 t)$ in (7) is small when Δp is small. Applying Taylor expansion to (12), we get

$$\alpha(t + \frac{1}{f_1}) - \alpha(t) = \Delta p(\int_0^{\frac{1}{f_1}} \chi(f_1 t) dt + \int_0^{\frac{1}{f_1}} \chi'(f_1 t) q(f_1 t) dt). \tag{13}$$

From (13) and (8), the ratio of frequency shift Δf to the parameter variance Δp can be expressed as

$$\frac{\Delta f}{\Delta p} = f_0 f_1 (\int_0^{\frac{1}{f_1}} \chi(f_1 t) dt + \int_0^{\frac{1}{f_1}} \chi'(f_1 t) q(f_1 t) dt). \tag{14}$$

The second part of the right hand side damps with Δp approaching 0, therefore the frequency sensitivity is

$$fs_p = \frac{\partial f}{\partial p} = \lim_{\Delta p \to 0} \frac{\Delta f}{\Delta p} = f_0 f_1 \int_0^{\frac{1}{f_1}} \chi(f_1 t) dt = f_0 \int_0^1 \chi(t_s) dt_s. \tag{15}$$

This equation only requires one integration for calculating the frequency sensitivity. The key element $\chi(t_s)$ in this equation is composed of two parts: $v_1^T(t)$ and $SF_p(t)$[5]. The former carries the phase sensitivity to equation perturbation which results in frequency variance, and the latter bears the equation sensitivity to parameter. Therefore, despite its simple expression, it contains all the elements for accurately calculating the frequency sensitivity.

Furthermore, for some kind of oscillator, even the analytical expression of the frequency sensitivity can be derived. This provides a direct design insight.

IV. Analytical Frequency Sensitivity for Ideal Three-Stage Ring Oscillator

In this section, an purely analytical frequency sensitivity is derived for an idealized three-stage ring oscillator. We demonstrate that the result is identical to a one-step derivation of a previously derived frequency expression [16]. The perfect conformity between these two expressions provides a persuasive proof of the correctness of our proposed FS-PPV.

Figure 1 shows the diagram of the idealized ring oscillator, in which the three inverters are assumed to switch abruptly at the input voltage equaling 0 and the output voltage is ± 1. All the resistors and capacitors are assumed identical ($R_1 = R_2 = R_3 = R, C_1 = C_2 = C_3 = C$), and we define $\tau = RC$.

In previous work [16], the analytical expressions of this oscillator's steady-state and PPV waveforms have been derived. And its period T ($T = \frac{1}{f_0}$) abides by $e^{\frac{T}{\tau}} = \varphi^6$, where φ is the Golden Mean $\frac{1+\sqrt{5}}{2}$.

Next, taking R_1 as the varying parameter, we derive the analytical expression for the frequency sensitivity.

Based on the derived steady-state and PPV waveforms[6], the key component $\chi(\cdot)$ in the frequency sensitivity expression (15) can be derived:

$$\chi(f_0 t) = \frac{1}{R_1^2 C_1} \cdot \begin{cases} -\frac{\tau}{\sqrt{5}} \varphi^3 & 0 \le t \le \frac{T}{6} \\ -\tau(\frac{2}{\sqrt{5}} - 1)\varphi^3 & \frac{T}{6} \le t \le \frac{T}{2} \\ (\frac{2}{\sqrt{5}} - 1)\tau\varphi^6 & \frac{T}{2} \le t \le \frac{2T}{3} \\ \frac{\tau}{\sqrt{5}} & \frac{2T}{3} \le t \le T \end{cases} . \tag{16}$$

Noticing that $f_0 = \frac{1}{T}$, $\tau = RC$, $e^{\frac{T}{\tau}} = \varphi^6$ and $\varphi = \frac{1+\sqrt{5}}{2}$, we can easily calculate the frequency sensitivity *wrt* R_1 and write it in a

[5] Only $SF_p(t)$ is included because it is based on the ODE equation, while for the general DAE case, another element $SQ_p(t)$ needs to be added.

[6] Refer to the paper [16] for the expressions.

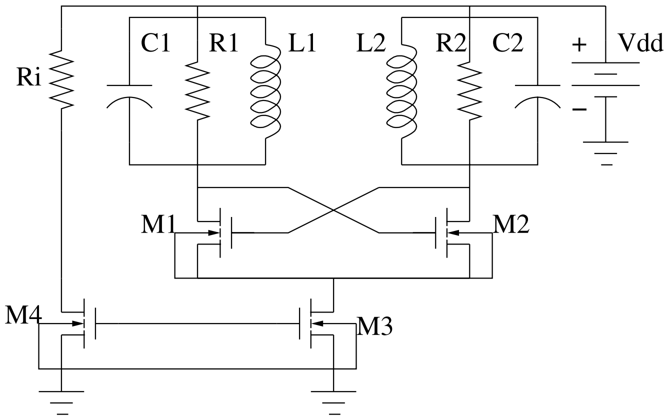

Fig. 2. Cross-coupled LC oscillator.

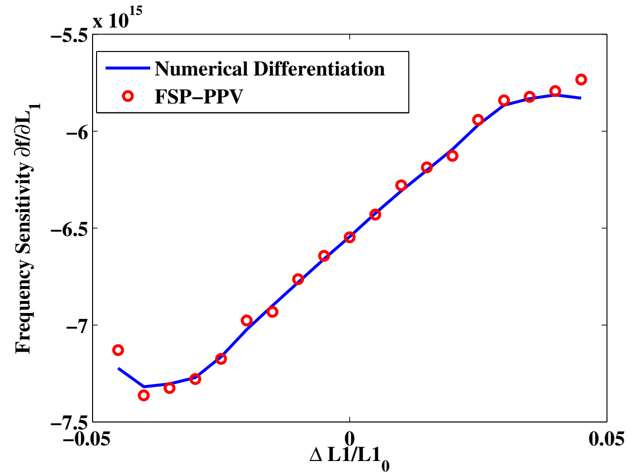

Fig. 3. Frequency Sensitivity from Two Methods.(LC OSC)

simple form:

$$fs_{R_1} = -\frac{f_0}{3R_1} = -\frac{f_0}{3R}. \quad (17)$$

This expression is based on our proposed FS-PPV. We can also derive one using the existing frequency expression [16]. Substituting T and τ with $\frac{1}{f_0}$ and RC in $e^{\frac{T}{\tau}} = \varphi^6$, we have

$$f_0 = \frac{1}{6RCln(\varphi)}. \quad (18)$$

Taking derivative of the f_0 *wrt* R, we get

$$\frac{\partial f}{\partial R} = -\frac{1}{6R^2Cln(\varphi)} = -\frac{f_0}{R}. \quad (19)$$

Because of the symmetry, each resister bears the same amount of the frequency sensitivity. Therefore, the frequency sensitivity *wrt* one resistor is

$$\tilde{fs}_R = -\frac{f_0}{3R}. \quad (20)$$

This expression is identical to (17), which we have derived using our proposed FS-PPV. The exact conformity consolidates our new method.

V. Numerical Results

In this section, we apply our proposed FS-PPV to two practical oscillators: cross-coupled LC and 3-stage ring oscillator. For each one, frequency sensitivity *wrt* one parameter over a range of this parameter is calculated, and the results are compared to those of the numerical differentiation method. Then, using the calculated frequency sensitivities, we map the statistical distribution from parameters to frequency with an analytical expression, for the simple case that all the considering parameters, assumed to be not correlated, are gaussianly distributed. The resulting PDFs have good matches to those of the Monte Carlo method, with speedups of more than 3000× at least.

A. Cross-Coupled LC Oscillator

Figure 2 shows the block diagram of the cross-coupled LC oscillator. We choose the nominal parameter $L1_0 = L2_0 = 2.4345e-7/(2\pi)H$ and $C1_0 = C2_0 = 4e-12/(2\pi)F$, making it oscillates at about 1GHz.

1) Frequency Sensitivity Calculation: We firstly use Harmonic Balance [5] to simulate the frequency and steady state waveform. The sensitivity function $SF_p(t)$ can be easily got after the steady state waveform is simulated. Secondly, PPV waveform $v_1^T(t)$ is extracted using the existing efficient tool [17]. Then, the frequency sensitivity is only a one-step integration of $\chi(t_s)$.

In this case, we calculate the sensitivity *wrt* L1, with $\Delta L1$ varying from $-0.05L1_0$ to $0.05L1_0$. As a validation, we estimate the frequency sensitivity at each parameter using the numerical differentiation. The results of both these two methods, plotted in Figure 3, have good matches to each other.

2) Parameter Variation Analysis: Frequency sensitivity *wrt* four parameters[7] (L1, L2, C1 and C2) are calculated at the nominal parameter. Afterward, we implement this technique for a purpose of parameter variation analysis. We only deal with the most simple case, in which all the parameters, assumed uncorrelated, are gaussianly distributed with their standard deviations being σ_{L1}, σ_{L2}, σ_{C1} and σ_{C2}. Based on a reasonable linearization approximation, the probability density function of frequency is also gaussianly distributed. Denoting the calculated frequency sensitivities as fs_{L1}, fs_{L2}, fs_{C1} and fs_{C2}, we can simply write the standard deviation of the frequency distribution as

$$\sigma_f = \sqrt{(fs_{L1}\sigma_{L1})^2 + (fs_{L2}\sigma_{L2})^2 + (fs_{C1}\sigma_{C1})^2 + (fs_{C2}\sigma_{C2})^2}. \quad (21)$$

To validate this approach, we do a Monte Carlo on the four parameters, whose relative standard deviations are chosen to be 0.01. About 3500 points are simulated. Figure 4 shows the histogram of the PDF of the frequency. The solid red line is the PDF plot using (21). The histogram is scaled for a good comparison. We demonstrate that the two PDFs have good matches to each other. This consolidates that our linearization approach is valid.

The time cost for calculating the frequency sensitivity equals one step of Monte Carlo simulation, so speedup of 3500× is easily achieved with the Monte Carlo taking only 3500 points. It's bigger if more samples are required for a better accuracy of the PDF. For more parameters being randomly distributed, our method only requires the frequency sensitivities *wrt* the extra parameter to be calculated, while using the Monte Carlo method we have to redo the whole simulation again taking far more points .

B. 3-Stage Ring Oscillator

Figure 5 shows the block diagram of the 3-stage ring oscillator. For this oscillator, we investigate the frequency sensitivity *wrt* the threshold voltages of the MOSFETs. At nominal value ($VTN1_0 = VTN2_0 = VTN3_0 = VTP1_0 = VTP2_0 = VTP3_0 = 0.3V$), the frequency is about 0.9GHz.

1) Frequency Sensitivity Calculation: Sensitivity *wrt* the threshold voltage of NMOS1 (VTN1) is calculated, with $\Delta VTN1$ varying from $-0.5VTN1_0$ to $0.5VTN1_0$. The result is compared to that of the numerical differentiation in Figure 6. The solid line, result of numerical differentiation, suffers zigzag because of the round off error, while the that of FS-PPV looks smooth.

2) Parameter Variation Analysis: Similarly in this case, we do a Monte Carlo simulation varying the threshold voltages of the six MOSFETs. Compared to Section V-A.2, more points (about 130000) are simulated, and a better histogram of the probability density function is obtained in Figure 7. We also derive an analytical expression for the PDF using the calculated frequency sensitivity and plot it with the histogram. We demonstrate that the two PDFs fit well.

[7]More parameters can be conveniently calculated without extra computational cost.

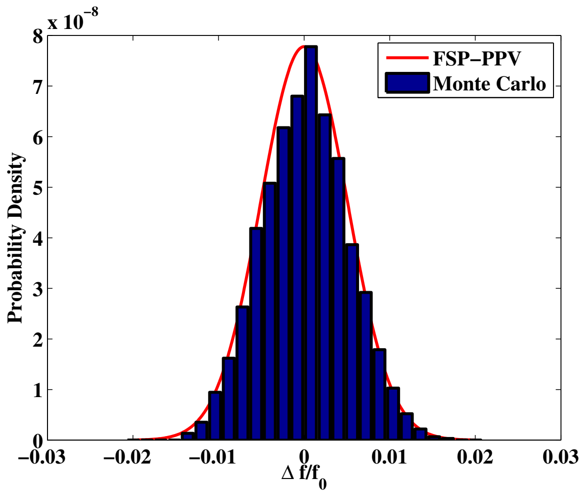

Fig. 4. Probability Density Function (PDF) from Two Methods.(LC OSC)

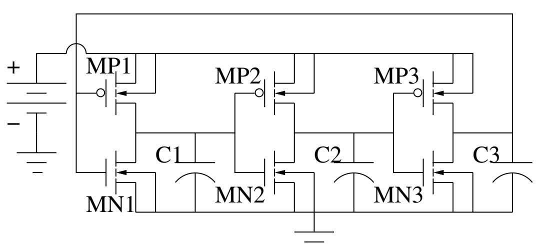

Fig. 5. 3-Stage Ring Oscillator.

VI. Conclusions

We have proposed a new method of obtaining frequency sensitivity analytically based on the PPV macromodel. The crucial feature of our FS-PPV method is it establish a link between the frequency sensitivity and the PPV with a nice simple expression, artfully implementing an existing efficient and mature technique (PPV macromodel) for keeping accuracy. It has been successfully implemented in the parameter variability problems, and its simple expression ensures that it can be potentially directed to more complicated parameter variability cases.

Acknowledgments

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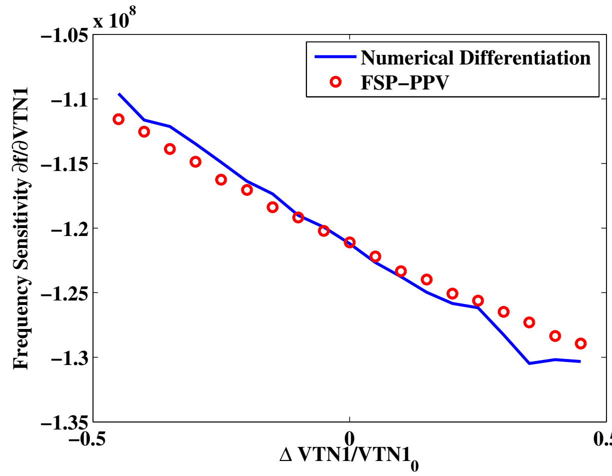

Fig. 6. Frequency Sensitivity from Two Methods.(RING OSC)

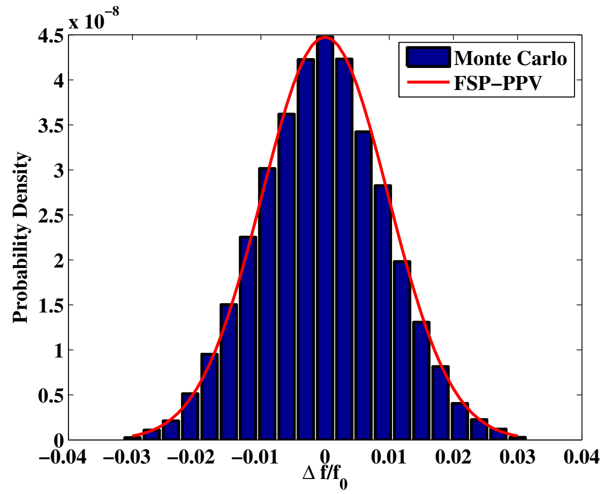

Fig. 7. Probability Density Function (PDF) from Two Methods.(RING OSC)

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